

# A medieval Arabic analysis of motion at an instant: the Avicennan sources to the *forma fluens/fluxus formae* debate

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**Abstract.** The *forma fluens/fluxus formae* debate concerns the question as to whether motion is something distinct from the body in motion, the flow of a distinct form identified with motion (*fluxus formae*), or nothing more than the successive states of the body in motion, the flow of some form found in one of Aristotle's ten categories (*forma fluens*). Although Albertus Magnus introduced this debate to the Latin West he drew his inspiration from Avicenna. This study argues that Albertus misclassified Avicenna's position, since Albertus could not conceptualize motion at an instant, whereas it is claimed here this was the very position Avicenna adopted. The paper includes an overview of Albertus's discussion and a brief survey of the Avicennan sources upon which Albertus drew. The heart of the paper treats Avicenna's analysis of motion at an instant. Avicenna's general argument was that since spatial points have no extremities, nothing in principle prevents a moving object from being at a spatial point for more than an instant, understood as a limit. It is then argued that Avicenna had the philosophical machinery to make sense of a limit, albeit not in mathematical terms, but in terms of an Aristotelian potential infinite.

The first and foremost topic of classical and medieval physics is the concept of motion (Grk. *kinēsis*, Arb. *ḥaraka*, Lat. *motio*). Within the complex of issues and problems associated with motion, the question 'in which category does motion itself belong?' occupied a position of considerable importance in scholastic natural philosophy. In the Latin tradition the problem was most typically framed in terms of whether motion should be characterized as a *forma fluens* or a *fluxus formae*. Albertus Magnus, it would seem, philosophically introduced this topic into Latin natural philosophy in his commentary on Text 4, Book 3 of Aristotle's *Physics*. Yet, as Albertus himself admits, the immediate historical origins of this debate are found in the writings of the Arabic philosophers Avicenna and Averroes.<sup>1</sup>

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1 Ahmad Hasnawi, in his recent study 'Le Statut catégorical du mouvement chez Avicenne: Contexte grec et postérité médiévale latine', in *De Zénon d'Élée à Poincaré: Recueil d'études en hommage à Roshdi Rashed*

This study starts by looking briefly at how Albertus frames this debate. He does so by investigating five approaches to the conceptualization of motion, which he claims to have drawn from Avicenna. Included within these five approaches are the *forma fluens* and *fluxus formae* characterizations of motion. Albertus maintains that Averroes espoused the former and Avicenna the latter position. I next quickly consider Avicenna's own historical catalogue of the various ways in which motion had been conceptualized. Albertus admits this was the precursor for his own characterizations. The heart of the paper considers to what extent Albertus has properly classified Avicenna's position concerning the characterization of motion. More specifically I suggest that Albertus and subsequent historians of science have misclassified Avicenna's characterization of motion. Of the five approaches, Albertus canvasses one which he virtually rejects out of hand as feeble or unsound (*sententia debilis*), for it takes motion to be a univocal term and so a category in itself that subsumes the various types of motion. I contend that the position is not as untenable as Albertus would have one believe. There are textual and philosophical grounds in the writing of Avicenna for thinking that he may have endorsed such a view.

Albertus addresses the question 'in which category does motion itself belong?' by delineating five ways of conceptualizing motion.<sup>2</sup> First, one may consider motion by reference to its mover. In this respect motion would fall within the category of action.<sup>3</sup> Second, one may consider motion by reference to what is moved or the mobile. In this respect motion would fall within the category of passion.<sup>4</sup> Third, one may consider motion by reference to its final end or goal (*finis et terminus*) or, to be more exact, to the successively acquired stages of a moving thing during its process towards its ultimate end. On this account motion is not considered to be essentially different from the end to which it is directed, but only differs from it in its mode of being. In this respect motion is identical in essence with the end it attains, so motion is an equivocal term, since motion's different ends fall under the distinct and irreducible categories of quantity, quality and place. This is the idea of a *forma fluens*, which Albertus ascribes to Averroes.<sup>5</sup> Fourth, one may again consider motion by reference to its end or goal. But instead of characterizing motion with respect to any of Aristotle's categories, one views it simply as a 'means leading to a categorical result' (*via ad rem praedicamenti*) or a 'principle leading to it' (*principium ad ipsam*). Motion, on this account, is a certain type of imperfection. It is not the complete or perfect possession of a given end but only a means to that end, and so does not strictly speaking exist, or at least does not fully exist. However, since the categories only contain things that exist, motion itself cannot

(ed. R. Morelon and A. Hasnawi), *Les Cahiers du MIDEO*, Volume 1, Louvain-Paris, 2004, 607–22, translates for the first time in a modern language the relevant Avicennan text, *aṭ-Ṭabī'īyāt, as-samā' aṭ-tabī'ī* II.2. Although Hasnawi's article and mine treat the same topic, the two articles do not repeat one another. Instead it is to be hoped they complement one another.

2 Albertus Magnus, *Opera Omnia* (ed. Wilhem Kübel), Volume 4, Part 1, *Physica* (ed. Paul Hossfeld), Aschendorff, 1987, III, 1, 3, 150–6.

3 Albertus Magnus, op. cit. (2), 150. 81–151. 5.

4 Albertus Magnus, op. cit. (2), 151. 5–15.

5 Albertus Magnus, op. cit. (2), 151. 16–52.

be subsumed under one of the recognized categories. In this respect motion is in a certain sense outside any categorical classification, so the only reality it has is as a means or way to a category. This is the idea of a *fluxus formae*, which Albertus ascribes to Avicenna.<sup>6</sup> Finally, one might consider motion as such to be an additional category in itself or per se (*motus est praedicamentum per se*). In this respect motion is a univocal term that identifies a category other than one of the canonical ten ultimate genera recognized by Aristotle in his *Categories*. As such this ‘new’ category subsumes, as its species, the various kinds of motion: quantitative, qualitative and local.<sup>7</sup>

There are textual reasons for thinking that Avicenna may well have classified motion under the fifth approach sketched above, namely as a univocal term identifying a form common to all the varieties of motion. I shall therefore merely consider Albertus’s arguments, both explicit and implicit, for rejecting the fifth approach. Albertus attacks this position by asserting that there simply is nothing in common or univocal (*secundum univocationem*) with respect to the various kinds of motion, so there cannot be some overarching genus that subsumes them as their species. Such a view would absurdly involve the subsumption of some ultimate genus under another.<sup>8</sup> One cannot help but feel that Albertus’s explicit critique against this thesis has begged the question. It simply denies what the thesis asserts, that there is some genus outside Aristotle’s canonical ten categories. Moreover, Albertus’s criticism seems to be nothing more than a blind appeal to the authority of Aristotle for it assumes that Aristotle has exhausted all the possible kinds of ultimate genera and that his ten categories are truly generic. If someone denied the philosophical and scientific value of Aristotle’s *Categories*, as Avicenna in fact does, then Albertus’s explicit criticism simply loses its force.

However, Albertus may also have had deeper, albeit implicit, philosophical concerns, as Annaliese Maier has suggested. She writes, ‘scholastic-Aristotelian philosophy was not able to comprehend non-static, successive phenomena and classify them in terms of its system of categories’.<sup>9</sup> As I understand Maier’s point, the concern lurking in the back of Albertus’s mind might be stated as follows. Assume that motion were its own special category, and so had a generic form corresponding with that category. In that case, an object in motion must possess this generic form of motion and so be actualized by this generic form as something in motion, at every instant that it is in motion. Here an ‘instant’ should be understood as a temporal point lacking any positive magnitude as opposed to, say, an extremely short period or duration of time. An example from one of

6 Albertus Magnus, op. cit. (2), 151. 53–85.

7 Albertus Magnus, op. cit. (2), 151. 86–152. 2.

8 Albertus Magnus, op. cit. (2), 149. 63–150. 8. Also at III, 1, 2, 149. 38–44, Albertus similarly argues that if motion were a category beyond Aristotle’s ten, then it would be truly *sui generis*, in that motion is found in other categories, namely the categories of quantity, quality and ‘where’ or place, whereas none of Aristotle’s other categories are found within one another, a conclusion that Albertus finds unacceptable. This argument, like the later one, presupposes that there are no genera other than Aristotle’s ten and that Aristotle’s ten categories are truly generic. But we shall see that this is the very question at stake.

9 ‘[D]ie aristotelisch-scholastische Philosophie hat keine Möglichkeit gehabt, nicht-statische, sukzessive Phänomene mit ihrem Kategorienapparat wirklich zu erfassen und ontologisch einzuordnen’; Annaliese Maier, *Die Vorläufer Galileis im 14. Jahrhundert*, Rome, 1966, 16; *idem*, ‘The nature of motion’, in *idem*, *On the Threshold of Exact Science* (tr. S. Sargent), Philadelphia, 1982, 30.

the ten canonical categories might make the issue clearer. Consider the category of quality under which the specific form ‘redness’ falls. If an object is red then at every instant during the time it is red it must have the form of redness; the form of redness explains the object’s being red. In similar fashion, if motion were its own special category then the same must hold: at every instant during the object’s motion the moving thing must have the form of motion. In Maier’s formulation this form must explain the moving thing’s non-static, successive states, while belonging to the moving thing at every instant during its motion.

But an immediate problem arises for such a project. Since the moving thing must possess this form at every instant it is in motion, yet since it is by possession of this form alone that the thing is in motion, the possession of such a form would appear to imply that there could be motion at an instant. However, inasmuch as the scholastic philosopher considered motion to be a gradual process, it must take time.<sup>10</sup> Thus one is left with the absurd consequence that something that requires a period of time to occur, namely motion, occurs in no time, namely at an instant. This problem, I believe, ultimately underwrites Albertus’s criticism of the characterization of motion as itself a category. For the Latin scholastics there appeared to be no way to comprehend a univocal and generic form of non-static, successive phenomena, at least not within the traditional Aristotelian categories. If Avicenna thinks that motion might be characterized as a special category beyond Aristotle’s ten, then this problem is the one he must address.

10 Although the following point does not appear in the course of Albertus’s discussion of motion as either a *forma fluens* or a *fluxus formae*, it anticipates arguments that Albertus gives later. Thus in a digression at IV, 2, 7, 249, 1–31 Albertus argues, loosely following Aristotle’s *Physics* IV 8, 215a, 29–216 a11, that a void would preclude the possibility of motion, for in a void an object would be moved in an ‘indivisible time’, or instant (*in tempore indivisibili*), which the reader is assumed to understand is impossible. The point is made more explicitly when Albertus comments on Aristotle’s account of time at IV, 3, 15, 287–9, where Albertus argues that all motion occurs in time. Albertus notes that some might think that generation, or the sun’s illuminating the horizon, or a candle’s illuminating a house, provide counter-examples to this position, since they seem to occur instantly. Albertus responds that these examples, in a sense, are cases of an end (*finis*) instead of motion. He then continues, ‘an end of a continuum is not a continuum, just as a point is not a line and an instant (*nunc*) is not a time. Similarly, the end of a continuous motion is neither a motion nor a part of motion’; *finis continui non est continuum, sicut punctus non est linea et nunc non est tempus. Similiter autem finis motus continui non est motus neque pars motus* (288. 74–7). Albertus’s most explicit statement of this point comes at VI.1.8, 461. 14–36 (in Volume 4, Part 2 of the *Opera Omnia*), where he argues ‘that nothing is moved or at rest in an instant’ (*quod nihil movetur neque quiescit in nunc*). He had previously argued (VI.1.4) that a certain fastness (*velocitas*) and slowness (*tarditas*) are associated with the motion of any contracted and determinate things, where one of the definitions of fastness is to pass over the same amount of space that a slower object passes over but in less time (*Secundum [de diffinitionibus velocis et tardi] est, quod velocius in minori tempore transeat aequale spatium cum tardo*, VI.1.3, 452, 13–14). He now has us consider two mobiles: *x*, which moves faster, and *y*, which moves slower. Next consider their purported motion at an instant (*nunc*). In this case *x*, the faster-moving object, should have moved over a slightly greater amount of space than *y*, the slower, but, given the definition of ‘fastness’, *x* should be able to cross the distance that *y* crossed in less time than *y*; however, *y* crossed the distance at an instant, but there is no time smaller than an instant. Consequently the assumption of motion at an instant entails an absurdity. Albertus clearly thinks that motion at an instant is impossible, yet if motion were its own category, motion at an instant should be possible.

Before turning to how Avicenna might respond to this difficulty, let us consider his own chapter on the category to which motion itself belongs in the *Physics* of his *Shifā'*, the text from which Albertus drew his inspiration.<sup>11</sup> Avicenna begins his discussion by observing that in general there are two broad ways to approach the question of the category to which motion itself belongs. Motion might indicate either a univocal term (*lafẓa bi-t-tawāṭu'*) or an equivocal term (*lafẓa bi-t-tashkīk*).<sup>12</sup> Among those who maintained that motion is an equivocal term, Avicenna relates that some said that 'when motion is related<sup>13</sup> to the cause *in* (*fī*) which it is [i.e. the matter], then it is the category of passion, or [when it is related] to the cause *from* (*'an*) which it is [i.e. the form or perhaps agent], then it is in the category of action'.<sup>14</sup> These two views correspond with Albertus's first two classifications of motion.

Others who viewed motion as an equivocal term, Avicenna continues, likened it to 'syncategorematic' terms such as 'existence' and 'accident'. 'Existence' can be said of all ten categories, though it does not belong to all of them in the exact same way. Similarly, 'accident' is said of all the categories excluding substance, though it is not a genus subsuming the nine categories of accidents. In this respect motion might be viewed as the flow (*sayyāl*), equivocally said to belong to various categories. Thus locomotion is the flow in the category of 'where', alteration is the flow in the category of 'quality' and augmentation and diminution are the flow in the category of 'quantity'. Yet flow is not some genus subsuming these categories as its species, but is said equivocally of each category.

Concerning this latter view, namely that motion is the flow within certain categories, Avicenna notes two further divisions. On the one hand, some made becoming-*x* and being-*x* specific differences within a category. For example, becoming black and being black are different species of the genus 'black'. On the other hand, others did not make becoming-*x* and being-*x* specific differences, and so the motion to a certain end, the coming to be of that end, is not specifically different from that end itself. This later view appears to be just the one Albertus described as a *forma fluens* and ascribed to Averroes.

Avicenna adds a third option for those who maintain that motion is an equivocal term, one to which he has already alluded. Instead of saying that motion is in the various categories in an equivocal sense, one might instead say that motion stands outside the categories and so might think of it as a 'syncategorematic' term, analogous to the terms 'existence' and 'unity', which do not subsume the categories as species. Avicenna hints that perhaps existence can be conceptually divided into perfect being and imperfect being, with motion corresponding to imperfect being.<sup>15</sup> This is the view that Albertus ascribes to Avicenna under the heading *fluxus formae*.

11 *Shifā'*, *aṭ-Ṭabī'iyāt*, *as-samā' aṭ-ṭabī'ī* (ed. S. Zāyed), Cairo, 1983, II.2; henceforth *aṭ-Ṭabī'iyāt*.

12 *Aṭ-Ṭabī'iyāt*, II.2, 93, 4–8.

13 The text's *nsyt* should clearly be corrected to read *nusibat*, which is the reading found in J. Āl-Yāsīn's edition of *Aṭ-Ṭabī'iyāt*, Beirut, 1996, 138.

14 *Aṭ-Ṭabī'iyāt*, II.2, 93, 13–14.

15 *Aṭ-Ṭabī'iyāt*, II.2, 94, 13–15. Although the term 'imperfection' is not used here, he mentions a 'first perfection' (*al-kamāl al-awwal*), which at *Physics* II.1, 84, 9–19 Avicenna identified with being at an intermediate point between the motions' termini *a quo* and *ad quem*, and so in a sense a first perfection is an

Avicenna finds that all three of these ways of conceptualizing motion as an equivocal term are philosophically wanting. He says that he detests (*astakrihu*) the second approach, namely that becoming-*x* and being-*x* are essentially identical, which appears to be the suggestion that motion is a *forma fluens* that Albertus ascribed to Averroes.<sup>16</sup> There simply can be no meaningful way, complains Avicenna, that they could be identical without in fact eliminating the very flow or motion that one wanted to explain.<sup>17</sup> If one says, for example, that blackening is an intensification of black, and if these two are identical, then the black is either something initially existing or not existing. Clearly, if the black is not existing, then there is no sense in which it is being intensified, whereas if the black is already there, then there cannot be a motion to it, since it already exists.<sup>18</sup>

The first and third views are also both philosophically problematic, complains Avicenna, and they share a similar problem. Again, according to the first view motion is an equivocal term in the sense that becoming black and being black, for instance, are specific differences of the sub-genus black; so, in general, motion would be said to exist in as many ways as there are different generic and specific kinds of existence within the categories of quantity, quality and 'where' or place. According to the third view, 'motion' is an equivocal term in just the way that terms such as 'existence', 'unity' and 'accident' are equivocal terms. Both views take motion to be in some sense analogous with existence. Yet one sees in Avicenna's complaint that there is a 'disanalogy' between existence and motion.<sup>19</sup> He observes that existence is a 'single concept, but it applies to the categories differently according to priority and posteriority; for existence belongs to the category of substance primarily, whereas it belongs to accidents secondarily'.<sup>20</sup> Here Avicenna appeals to what G. E. L. Owen later calls a 'focal meaning', or an instance of *pros hen* equivocation, the suggestion that when a term is used analogously, then there is some primary meaning from which its other applications are in some sense derived.<sup>21</sup> For example, the existence that substances have provides the primary sense of existence, whereas the existence that accidents have is derived secondarily and analogously from one's understanding of a substance's existence. With respect to motion, however, there is no analogous priority relation between the various kinds of motion.<sup>22</sup> Locomotion, for example, does not provide the focal meaning of

imperfection of a mobile with respect to its second perfection, what will be the ultimate completion of the motion. This point is discussed more fully below.

16 *Aṭ-Ṭabī'īyāt*, II.2, 94. 16–17.

17 *Aṭ-Ṭabī'īyāt*, II.2, 94. 17–95. 6.

18 According to Avicenna the confusion arises since the term 'black', which one takes to be intensified in the process of blackening, actually indicates a resemblance relation among various individual shades of black (or darkness) to some obscure paradigm case of black. Thus one mistakenly thinks that a perduring black is being intensified, when in fact it is merely a certain substance or subject that is being intensified with respect to black, gradually taking on a spectrum of shades or different species of black; see *aṭ-Ṭabī'īyāt*, II.2, 95.9–10.

19 *Aṭ-Ṭabī'īyāt*, II.2, 95.10–96.10.

20 *Aṭ-Ṭabī'īyāt*, II.2, 96.1–3.

21 G. E. L. Owen, 'Logic and metaphysics in some earlier works of Aristotle', in *Logic, Science and Dialectic: Collected Papers in Greek Philosophy*, Ithaca, NY, 1986, 180–99, 184–99.

22 Avicenna is aware of Aristotle's arguments at *Physics* VIII 7, 260a, 26–261a, 12 that in one sense locomotion is primary. Thus Avicenna admits that one type of motion might be the reason (*sabab*) that

alteration or augmentation, nor conversely do these others provide it for locomotion. Thus, according to Avicenna, the analogy between ‘motion’ and ‘existence’ fails, and so one cannot in a principled way say that motion belongs to various different categories equivocally or analogically as the term ‘existence’ does.

Let us return to Avicenna’s initial broad division of the ways of characterizing motion. If ‘motion’ is not an equivocal term, then it must be a univocal term. Here Avicenna claims that the stalwart Aristotelian, clinging to Aristotle’s canonical ten categories, had best relate motion to the category of passion, for motion is defined in terms of potentiality and so, loosely, passivity.<sup>23</sup> In this case, however, the defenders of univocity must take the category of passion so broadly that the category of passion itself takes on an equivocal meaning. Avicenna simply complains in this case that if one is willing to give up univocity with respect to the category of passion in order to apply it to motion, one could just as well give up univocity with respect to motion.<sup>24</sup>

In order to appreciate Avicenna’s own position concerning motion one must bear in mind his attitude towards Aristotle’s *Categories*. In fact Avicenna is quite sceptical of the philosophical value of this work. Thus he states that ‘we are not all that inflexible in preserving the commonly accepted canon, namely that the genera are ten, and that each of them is truly generic and that there is nothing outside of them’.<sup>25</sup> Thus, concludes Avicenna, motion is a univocal term that is related to ‘the category whatever [it] is’ (*al-maqūla mā huwa*).<sup>26</sup>

Avicenna obliquely explains this cryptic comment – ‘the category whatever [it] is’ – at the beginning of the next chapter (*Physics* II.3). There he again summarizes the various ways of characterizing motion’s relation to the categories and suggests that motion is related to the category of substance, clearly not as a substance

another type of motion exists, for example the Sun’s motion in the heavens (i.e. a local motion) brings about a change in the Earth’s temperature (i.e. a qualitative change). Still, local motion’s having one description is not the cause (*‘illa*) of the other motions’ having their description. ‘So, locomotion’s being in this description [i.e. being a motion] is not a cause for alteration’s being in this description, although the existence of locomotion can be a reason for the existence of the alteration’ (*aṭ-Ṭabī’iyāt*, II.2, 96.4–5).

23 Ernest J. McCullough has wrongly maintained that in Avicenna’s considered opinion motion must be identified with the category of passion. He reasons that as in the end Avicenna does explicitly say, ‘Now the doctrines that we have identified are invalidated, and we did not accept them’, so ‘the truth alone remains: the first doctrine’ (*aṭ-Ṭabī’iyāt*, II.2, 97. 13), which McCullough identifies with making motion belong to the category of passion. In fact the first doctrine is simply that motion is a univocal term, and that some who maintain this view make motion univocally the category of passion. Avicenna’s position is simply that motion is a univocal term and that if one wants to retain the Aristotelian categorical schema, the category of passion is the best candidate. However, Avicenna himself sees no reason to cling to Aristotle’s categories. See Ernest J. McCullough, ‘St. Albert on motion as *Forma fluens* and *Fluxus formae*’, in *Albertus Magnus and the Science* (ed. J. Weisheipl), Toronto, 1980, 129–53, in §B.

24 *Aṭ-Ṭabī’iyāt*, II.2, 95. 14–16.

25 *Aṭ-Ṭabī’iyāt*, II.2, 97. 10–12.

26 Alternatively, and perhaps more literally, *al-maqūla mā huwa* can be translated as ‘the category *what is*’. ‘What is’ may be an oblique reference to the category of substance, and given what Avicenna says in the next chapter, there is some textual evidence for this reading (see what follows in the body of the paper). Still, such a suggestion is at least problematic given Avicenna’s immediately preceding remarks that if one is going to classify motion within the Aristotelian categorical scheme, one would do best to put it in the category of passion, not substance. Unfortunately, Hasnawi, in his translation in ‘Le statut catégorical du mouvement’ (op. cit. (1)), does not translate this last sentence of Avicenna’s text.

itself, but as an intensification or diminution of a substance with respect to some category.<sup>27</sup> For example, if I spend too much time in the sun I will go from a pale pink to ‘lobster red’. Here a substance, myself, has undergone intensification with respect to the category of quality; namely, the initial form of redness that I had was replaced with a more intense or brighter form of redness. Avicenna’s suggestion, then, appears to be that the form of motion belongs to a substance just as any other accidental, generic form does. There is a significant exception, however, in that it terminates in the substance’s possessing some new form with respect to one of the categories susceptible to motion, such as quantity, quality, place or ‘where’ and, for Avicenna, the additional category of position. Thus Albertus was correct in describing Avicenna’s account of motion as a ‘means leading to a categorical result’. But he was wrong in thinking that motion for Avicenna does not indicate a univocal term identifying a generic form of motion distinct from one of Aristotle’s ten categories. Again, there is nothing untoward in Avicenna’s position given the doubts that he has about the philosophical and scientific merit of Aristotle’s *Categories*. In short, it would seem that for Avicenna motion may well be a univocal term that stands outside Aristotle’s traditional categorical scheme, yet in some sense subsumes the various kinds of motion.

Here, then, we return to the initial problem implicit in Albertus’s critique of the treatment of motion as a univocal term. If motion is some univocal term or generic form, then (1) ‘what is this common form of motion?’ and (2), more importantly, ‘how can there be a univocal and generic form of motion that characterizes a non-static, successive phenomenon?’ It is this second point, I claimed, that lies at the heart of Albertus’s concern. The trouble is once again that a form must belong to a subject at every instant it has that form. So if there were a univocal form of motion, then there could be motion at an instant, so not requiring a period of time, yet motion seems to require a period of time. Thus to posit a generic and univocal form of motion seems to imply the contradiction that something can occur without taking any time, but can only occur during a time. The remainder of this paper suggests that Avicenna had the philosophical machinery to handle this problem of characterizing the univocal form of motion.

As with any characterization of motion during the Middle Ages, Avicenna begins with Aristotle’s definition of motion as ‘the *entelekheia* of what is potential insofar as it is such’ (*Phys.* III 1, 201a10–11). In similar fashion, Avicenna, following the Arabic translation of Aristotle’s *Physics*, defines motion as ‘the first perfection (*kamāl*) belonging to what is in potency inasmuch as it is in potency’.<sup>28</sup> Two points must be noted

27 ‘Let us start from the beginning, even if it perhaps contains a repetition of some of what was said. We say that our doctrine that a motion is in such a category could be understood in four senses. The first of these is that the category is a true subject for [the motion] subsisting in itself. The second is that even if the category is not the substantial subject belonging to [the motion], [the motion] is realized in the substance through [the category’s] intermediacy, when [motion] is primarily found in [the category], just as smoothness belongs to substance only through the intermediacy of the surface. The third is that the category is a genus and species belonging to [the motion]. The fourth is that the substance is moved from a species belonging to that category to another species, and from kind to kind [within that category]. The sense that we adopt is the last one [i.e. the fourth].’ (*At-Tabrī‘iyāt*, II.3, 98, 8, 9).

28 *At-Tabrī‘iyāt*, II.1, 83.5. The Arabic translation of Aristotle’s definition is ‘motion is the perfection of what is in potency in what is such; *al-ḥaraka hiya kamālu mā bi-l-qūwa bi-mā huwa ka-dhālika*.’

about Avicenna's definition: first, its use of *kamāl* as a translation of the Greek *entelekheia*, and second, its introduction of 'first' to modify *entelekheia* or *kamāl*.

Consider the latter point first. The proper way to understand the key notion of *entelekheia* in Aristotle's definition of motion was a matter of considerable interpretative dispute among Aristotle's later Hellenistic commentators.<sup>29</sup> As a means of clarifying this notion they appealed to Aristotle's additional comments in the *De Anima*, where Aristotle distinguished between first and second *entelekheiai*.<sup>30</sup> The distinction was, for example, between one who knows a language but is not currently using it – a first *entelekheia* – and one who is currently using a language – a second *entelekheia*. With respect to motion there was universal agreement that motion's second *entelekheia* referred to the final end or state of perfection reached at the termination of the motion, whereas its first *entelekheia* referred to the intermediate state of the mobile between its initial state of potentiality and its final state of perfection or its second *entelekheia*. Yet there were divergences among Aristotle's Greek commentators as to how a first *entelekheia* referred to this intermediate state. For instance, Alexander of Aphrodisias glossed 'first *entelekheia*' with the apparently non-process term *teleiotēs*, i.e. as perfection or completion of an intermediate state.<sup>31</sup> In contrast, Themistius understood first *entelekheia* as inherently involving a process, for although he also used *teleiotēs* as a gloss for *entelekheia*, he understood *teleiotēs* so elastically that in one of its significations, namely with respect to a first *entelekheia*, it indicates a gradual transition or process, as is witnessed by his further gloss of 'first *entelekheia*' with the clearly process term *poreia*, a passage, procession or traversal to a final perfection.<sup>32</sup> In short, at least one of the classical issues surrounding Aristotle's definition of *kinēsis*, or motion, is whether the key term in its definition, *entelekheia*, was a process term or not.<sup>33</sup>

Aristotle's definition of motion is intended to provide the most basic account of what a process is. So, on the one hand, if *entelekheia* is a process term, then it is easy to see how Aristotle's definition describes a process. But it apparently does so at the cost of making Aristotle's definition circular. For if *kinēsis* is the most basic process term, then one must already know what a *kinēsis* is in order to understand what an *entelekheia* is. On the other hand, if *entelekheia* is not a process term, then one avoids circularity,

29 For an excellent discussion of the understanding of *entelekheia* by the Greek commentators, both Peripatetics and Neoplatonists, especially as it would relate to Avicenna's own thought and how it was transmitted into Arabic, see Robert Wisnovsky, *Avicenna's Metaphysics in Context*, Ithaca, NY, 2003, Part I.

30 Aristotle, *De anima* II 1, 412a, 22–8.

31 Since Alexander's *Physics* commentary is lost one must rely on what he does say about *entelekheia* in his extant works as well as the testimonia and fragments found in other commentators; see Alexander, *De anima*, in *Praeter commentaria scripta minora, Commentaria in Aristotelem Graeca* (ed. I. Bruns), Pars I, Volume 2, Berlin, 1887, 8, 12–9, 12, 16, 4–18 and 103, 3–4; also see Simplicius, *In Aristotelis Physicorum libros quattuor priores commentaria* (ed. H. Diels), Volume 9, Berlin, 1882, 414, 29–415, 24 and 416, 27–417, 28.

32 For Themistius see *In Aristotelis Physica Paraphrasis* (ed. H. Schenkl), Volume 5, Berlin, 1900, 70, 5–13.

33 For contemporary interpretations of Aristotle on this point see J. L. Ackrill, 'Aristotle's distinction between *Energeia* and *Kinesis*', in *New Essays on Plato and Aristotle* (ed. R. Bambrough), New York, 1965, 121–41, who leans towards a process interpretation; or L. A. Kosman, 'Aristotle's definition of motion', *Phronesis* (1969), 14, 40–62; and Mary Louise Gill, 'Aristotle's theory of causal action in *Physics* III 3', *Phronesis* (1980), 25, 129–47, both of whom take a non-process reading of Aristotle's *entelekheia*.

but it is no longer clear how Aristotle's definition of *kinēsis* actually describes a process, which is the very thing the definition is supposed to make clear.

These options roughly summarize one aspect of the debate surrounding motion as it reached Avicenna. This leads to the second point, namely how Avicenna understood *entelekheia*, i.e. *kamāl*, in Aristotle's definition of motion. On the question of whether *entelekheia* should be understood as a process term or as a non-process term, Avicenna clearly sided with the latter: motion could not be defined in terms of a process. For Avicenna the definition of motion was intended to provide the natural philosopher with his most basic kinematic account. As such the definition could not explicitly or implicitly employ some other kinematic or process term. 'Passage', 'procession', 'traversal' and the like (Avicenna mentions *zawāl* and *sulūk*, which are most likely translations of Themistius's *poreia*) are all synonyms for motion.<sup>34</sup> In fact, observes Avicenna, they refer to a particular kind of motion, namely exchange of place. So to define motion in process terms is either to give a synonym for the *definiendum* or to define the more general in terms of the more specific.<sup>35</sup> In either case the purported definition would lack explanatory power, and so should be rejected as a proper scientific or philosophical definition.

The issue then arises of the understanding of *entelekheia* as the actuality or perfection of the mobile in the intermediate state or states. Avicenna continues that this actuality or perfection might refer either to the motion as some continuous magnitude extending across the interval, whether the whole of the interval or successive, extended increments or parts that constitute the whole motion, or to the actuality of the mobile at non-extended points within the interval. For Avicenna it is simply impossible that motion as it exists in the world is ever concretely actualized as an extended magnitude, whether the whole of the interval or successive, extended parts. Rather, motion as something extended exists as actual only in the estimative faculty or imagination of an individual observing the motion.

If one means by motion the continuous intelligible thing belonging to that which is moved between the starting and ending points, then that [account of motion] absolutely does not occur to that which is moved while it is between the starting and ending points; rather, one supposes that this has occurred in some way only when that which is moved is at the endpoint, but at that place this continuous intelligible has ceased with regard to existence. So how could it have a real occurrence in existence? On the contrary, this is something that really does not itself subsist in concrete particulars. It leaves an impression on the imagination only because its form subsists in the mind by reason of the relation between that which is moved and two places: the place which it leaves and the place which it reaches. Or it leaves an impression in the imagination because the form of that which is moved, having an occurrence in a place and a proximity and remoteness to [other] bodies, has been imprinted on [the imagination], and thereupon another form follows it from the point of view of sensory perception by its occurring in another place and a different proximity and remoteness. Thus one is aware of the two forms together as being a single form belonging to a motion, but [this form] does not occur subsisting in existence the way it does in the mind, since with respect to existence that which is moved does not occur in the two limits together nor does the state [of the motion] which is between the two have a subsisting existence.<sup>36</sup>

34 *Aṭ-Ṭabīrīyāt*, II.1, 83, 14–15.

35 *Aṭ-Ṭabīrīyāt*, II.1, 83, 15–17.

36 *Aṭ-Ṭabīrīyāt*, II.1, 84, 1–8.

An example will clarify Avicenna's point. If one considers the motion of a ball between two spatially separated places,  $x$  and  $y$ , the ball's motion is never fully actualized as a continuous interval extending from  $x$  to  $y$  in the way, for example, that the spatial distance between  $x$  and  $y$  exists as fully actual all at once. Rather, during the motion the ball exists at one spot along this interval and then ceases to exist at that spot, only to exist in another spot, and so on. Only after the motion is completed, or some part of it is completed, does one have an idea of the motion as extending through the interval. Consequently this type of actuality, motion as something extended, is only a *res intentionis* or an intentional object. So it has no extra-mental reality in concrete particulars. The account of motion that the natural philosopher wants, though, is how motion exists in the extra-mental world, not merely in the intellect. Thus, according to Avicenna, motion must be the first actuality or perfection of a potential belonging to a moving thing at the non-extended, intermediate points during the motion.<sup>37</sup> For Avicenna, then, the extra-mental reality of motion reduces to the existence of motion at an instant.<sup>38</sup>

This is an extraordinary claim for any medieval natural philosopher to make, especially in light of the difficulty suggested when developing Albertus's criticism of motion's being a univocal term. Thus we must ask whether Avicenna had the philosophical wherewithal to tackle Albertus's concern. For Avicenna, any account of motion at an instant had to steer a course between two philosophically unacceptable alternatives: one imposing an atomic structure on space and time, the other making the mobile come to rest at an intermediate point.<sup>39</sup> Atomism makes time and space a composite or aggregate of a finite set of discrete, extended, minimal temporal and spatial units. Since Avicenna had explicit philosophical reasons for denying that space and time had atomic structures, if his overall system is to be consistent he must avoid explaining motion at an instant in atomic terms.<sup>40</sup>

37 *Aṭ-Ṭabī'iyāt*, II.1, 84, 9–19.

38 For a detailed discussion of Avicenna's distinction between motion as it exists in the intellect and as it exists in the world, as well as an initial introduction to Avicenna's conception of a motion at an instant, see Ahmad Hasnawi, 'La définition de mouvement dans la *Physique* du *Šifā'* d'Avicenne', *Arabic Science and Philosophy* (2001), 11, 219–55; and for a discussion Avicenna's view of motion at an instant in his dynamics see Marwan Rashed, 'Natural philosophy', in *The Cambridge Companion to Arabic Philosophy* (ed. P. Adamson and R. C. Taylor) Cambridge, 2005, 287–307, 295–302.

39 For the sake of completeness I should also mention an-Nazzām's theory of the 'leap' (*tafra*), which in salient ways is a hybrid of atomic and continuous theories of magnitudes. In this theory an object moves from one spatial position to another through a series of discrete leaps without being in the intervening places, and so is akin to an atomic account of motion. But the space over which the object leaps is continuous space, so the theory is akin to a continuous theory of magnitude. Avicenna has very little patience with the theory of the leap and says little about it. He simply calls it 'repugnant' (*shanā'a*) (*aṭ-Ṭabī'iyāt*, III.3, 188.2). He seems to think that the articulation of any alternative theory that can deal with Zeno-like paradoxes of motion, which he claims the theory of the leap aimed to address, is sufficient evidence against the theory of the leap.

40 See *aṭ-Ṭabī'iyāt*, II.11, 159, 9–16 for Avicenna's initial argument against temporal atomism. For a translation and brief analysis of Avicenna's general arguments against atomism from his *Shifā'* see Paul Lettinck, 'Ibn Sīnā on atomism', *Al-Shajarah* (1999) 4, 1–51; and likewise for a brief overview see F. A. Shamsi, 'Ibn Sīnā's argument against atomicity', in *Uluslararası İbni Sīnā sempozyumu Bildirileri* (ed. M. Cunbur and O. Doğan), Ankara, 1984, 479–94. For a discussion of various versions of temporal atomism

If, in contrast, one views time and space as continuous, as does Avicenna, then there are reasons for thinking that if the mobile is actually at an intermediate point, it must come to rest at that given point and remain there for some brief period of time, such that there would not in fact be motion at an instant. Aristotle suggested the argument in a notoriously difficult passage at VIII 8 of his *Physics*.<sup>41</sup> The position Avicenna wants to avoid is roughly this: if a moving thing is actually (not merely potentially) at some point during its motion, as Avicenna maintains, then there must be the instant when it comes to that point and the instant when it leaves that point. ‘To come to be at some point’ and ‘to leave that same point’ appear to be contradictory predicates, or at the very least contraries. Thus a mobile cannot simultaneously come to be actually at a point when it is also actually leaving that point without violating the law of non-contradiction. At the very least, it cannot do so without violating the principle that two contrary predicates cannot be simultaneously true of one and the same thing, though both might be simultaneously false. Thus there must be two distinct instants: one of arrival and another of departure. One is assuming that time is continuous, and on a continuum no two points are immediately adjacent to one another. It follows that the two instants, or temporal points, cannot be immediately adjacent. Instead there must be a certain increment of time between the two instants. As such the mobile would be at rest at that spot during that time.

Even setting aside the pressing issue of whether such a view would mean that the traversal of a finite spatial magnitude would take an infinite amount of time, the argument clearly implies that the mobile cannot actually be at a point for an instant during its motion, so there cannot be motion at an instant. Aristotle’s own response was that the mobile is not actually but only potentially at the various intermediate points during the motion.<sup>42</sup> However, since Avicenna identifies motion with the instantaneous actuality of the mobile at its intermediate points, he must find another solution.<sup>43</sup>

within the Islamic medieval world, and Avicenna’s specific refutations of these accounts, see Jon McGinnis, ‘The topology of time: an analysis of medieval Islamic accounts of discrete and continuous time’, *Modern Schoolman* (2003), 81, 5–25.

41 *Physics* VIII 8, 262a, 119–b8.

42 *Physics* VIII 8, 262a, 21–5.

43 Avicenna, like Aristotle, denies that there are an infinite number of actualized points in a continuum. For both thinkers the points only exist potentially in the continuous magnitude and only become actual through pointing (*ishāra*) at a spot, something’s touching (*mumāssa*) or being parallel with (*muwāzā*) the spot or in some way dividing the continuous magnitude; see *aṭ-Ṭabī’īyāt*, III.3, 184, 6–16; and for a simplified but expanded version see his *Letter to the Vizier Abū Sa’d*, in *Lettre au Vizir Abū Sa’d* (ed. and tr. (in French) Yahya Michot), *Sagesses musulmanes* 4, Beirut, 2000, 37–53, ‘On the body, its dimensions and continuity’. It is unclear whether Avicenna believes that the mobile’s motion over a continuum successively actualizes the potentially infinite number of points in the continuum. Inasmuch as the mobile ‘touches’ and ‘is parallel’ with the potentially infinite number of points on the continuum, and inasmuch as ‘touching’ and ‘being parallel’ are means of actualizing the potential points in a continuum, it seems that the potentially infinite number of points would have successively, each for an instant, been actualized and then passed back into potentiality. Such a position initially seems improbable; an infinite would have been actualized, but according to Avicenna an actual infinite is impossible. But the issue is more complex. For Avicenna the impossible sense of an ‘actual infinite’ is that in which all its members simultaneously coexist. But here, though a potentially infinite number of points would have been successively actualized, all the infinite points are never simultaneously actualized. The situation is analogous to the infinite, or perhaps better ‘indefinite’, number of past days that the cosmos’s

Briefly stated, Avicenna avoids the two pitfalls of atomism and of making the mobile come to rest by allowing the mobile to exist actually in some intermediate point for a durationless instant (*ān*). For since the spatial point itself does not have two distinct extremities, there is in principle no reason why there must be two distinct instants, one where the moving object comes to be at the spatial point and another where it ceases to be at that point. Consequently the mobile need only be at that point for a durationless instant. Thus Avicenna says of the actuality of motion as it exists in the world,

The account [of motion] existing in act to which it is appropriate that the name ['motion'] apply and is the motion that is found in the mobile [as opposed to merely being in the intellect] is the intermediate state [of that which is moved] when it is neither in the first limit of the spatial magnitude nor occurring at its end. Indeed, it is in an intermediate limiting point (*ḥadd mutawassit*) in such a way that it does not exist in even one of the instants that occur during the duration of its passage into act as something fully realized at that limiting point [i.e. as something at rest at that point], in which case its being fully realized at any moment (*waqt*) you posit would be as something dividing (*qāṭi'*) some spatial magnitude while it is still in the act of dividing. This is the form of motion found in the mobile, namely, an intermediateness between the assumed starting point (*mabda'*) and the end inasmuch as any limiting point in which neither its before nor after is posited is not like the two points of two limits (*ḥaddayī tarafaynī*), where concerning [the limiting point] there is not the assumption that its before nor its after exist in it. Thus this intermediateness is *the form of the motion and is a single description* that necessarily accompanies the mobile and is not subject to change in any way as long as it is a mobile.<sup>44</sup>

pre-eternity entails, a position that Avicenna decidedly holds. At II.3 of the *Physics* of his *Shifā'* Avicenna implicitly suggests that in alteration, a type of continuous motion, a potentially infinite number of accidental species of quality are successively actualized: 'if the substance does not remain with the intensification, then the intensification will have brought about another substance and likewise in each instant posited for the intensification another substance would come to be, when the first had ceased, but between a substance and a substance there would be a potentially infinite [number of] substantial species, as is [the case] with qualities' (*aṭ-Tabrī'iyāt*, II.3, 98.15–17; added emphasis). Here Avicenna is arguing against a potentially infinite number of substances' being successively actualized, which he contrasts with the potentially infinite number of accidents being successively actualized. Thus he implicitly seems to suggest that such a successive actualization of accidents occurs. Again, though, it is not clear what Avicenna's ultimate position is on this point. Whatever the case, there is still a sense that for Avicenna the moving object is actually at the potential points in a continuum, albeit only for an instant, and so Aristotle's difficulty remains.

<sup>44</sup> *Aṭ-Tabrī'iyāt*, II.1, 84.10–14; added emphasis. In addition to the philosophical reasons treated above, the Latin translation of this passage – which is again where Avicenna provides his formal analysis of motion at an instant – almost certainly contributed to Albertus's and other Latin Schoolmen's failure or disinclination to attribute to Avicenna the concept of motion at an instant. Although most of the other relevant Latin passages that I have consulted are either relatively faithful translations or at the least close paraphrases, this passage is a decided exception. The Latin reads, *Intellectus vero quo habet esse in actu, cui nomen ['motio'] accidit ut sit motus qui est in mobili, ipse est dispositio eius media quamdiu non est in principio motionis nec pervenit adhuc ad finem eius; sed est in spatio medio sic quod in illo spatio, non est in aliquo instantium quod habuit in tempore sui exitus ad effectum. Ergo suum esse, quandocumque notaveris, erit praeteriens locum quamdiu fuerit in transitu, et hoc est forma motus quae est in mobili, et hoc est medietas inter principium propositum et finem, scilicet ut, in quo puncto posueris, non sit in eo amplius sicut nec antea nec post, non sicut duo termini extremitatum. Et haec medietas est forma motus, et est proprietas una quae comitatur mobile et non discedit ab eo quamdiu est mobile*; Indeed the intellect by which *x* has being in act – to which the name ['motion'] belongs, since being in the mobile, it is moved – is *x*'s intermediary state as long as it is neither at the beginning of the motion nor has yet arrived at its end; rather, it is in the intermediary space (*in spatio medio = fī ḥadd mutawassit*) such that in that space it is not at one of the instants that it had during the time of its going to completion. Therefore, its existence, whenever you take

We shall try to clarify Avicenna's meaning here. He is considering the mobile only insofar as it is at, or passes through, an intermediate point. Since spatial points do not have extremities, the motion at or through an intermediate spatial point does not have a corresponding 'before' and 'after' at that point. In other words, a spatial point has no extremity where the mobile can be said to have arrived at the spatial point and no different extremity where the mobile can be said to have left that point. Consequently, so Avicenna argues, there need be no distinct temporal point corresponding to the moment when the moving object has come to be at some intermediate spatial point, and no other distinct temporal point when it has ceased to be at that spot. This is because the spatial point itself has no such extremities or limits to which the temporal points must correspond. Consequently the moving thing need not be at some spatial point for more than an instant, so does not remain at rest there. But neither is the instant some minimal atomic unit of time, since it has no positive magnitude. Therefore the moving thing is literally in motion at that point for only an instant.<sup>45</sup>

So the challenge before Avicenna is to make sense of the notion of an instant, an issue all the more pressing in view of the concern we suggested underlies Albertus's objection. An instant does not exist gradually over a period of time, whereas motion does seem to exist gradually over a period of time. At II.12 of his *Physics*

note of it, will be something passing through a place as long as it is in transit. This is the form of motion that is in the mobile, i.e., the intermediateness between the posited beginning and end, namely that in whichever point you posit it, it is not in it any more like before and after, nor like the two termini of the extremes. This intermediateness is the form of motion and is a single property that accompanies the mobile and is not separated from it as long as it is a mobile.

*Liber primus naturalium* (ed. S. van Riet, J. Janssens and A. Allard) Avicenna latinus, Louvain-la-Neuve, forthcoming, 155, 22–33.

The Latin Avicenna, far from providing an analysis of motion at an instant, appears to deny it outright and rather makes the form of motion a certain type of directedness towards the end. Certainly 'directedness' is part of the form of motion for Avicenna, and elsewhere in his *Physics* Avicenna describes motion as 'taking itself toward' (*at-tawajjuh ilayhi*) its end. Yet even if motion's directedness is implicit here, the idea of motion at an instant, which Avicenna is trying here to make intelligible, is wholly lost in the translation. The translator has rendered the Arabic *ḥadd*, which elsewhere he correctly translates as *extremum*, with the Latin *spatium* (space, room, extent). But the Arabic *ḥadd*, literally a limit, edge or boundary point, simply cannot bear the sense of the Latin *spatium*. Moreover, where Avicenna explicitly states that the intermediary point where there is motion at an instant is not like an extended point, i.e. some atomic unit that has two limits, the translation denies that the motion is at a point at all, i.e. it is not at any point that is like the beginning or ending point. Although a careful comparison between the Arabic and Latin Avicennas is needed, limits of space and the extent of the topic preclude such a comparison here. It is hoped that this gives enough indication of the changes and implications for the history of science of the Latin translation of Avicenna's *Physics*.

<sup>45</sup> Although Avicenna never directly treats Albertus's objection to motion at an instant based on fastness and slowness, we now have the materials to see how he might have responded. Albertus assumed that fastness and slowness, understood as distance covered over the time, necessarily apply to every contracted and determinate motion; he then used this assumption to generate his absurdity that a faster-moving object would traverse a certain amount of space in less than an instant. Avicenna can respond that fastness and slowness only apply to motions that have occurred over a certain amount of space or distance during a certain period of time, but the form of this motion, as something extending over a distance, Avicenna has claimed, only exists in the estimative faculty, not in the world. In other words, fastness and slowness simply do not apply to motion at a point at an instant, which is the form of motion in the world and is the same for every motion, which is what Avicenna wanted to prove.

Avicenna explains two different senses of ‘non-gradual’ by which to understand an instant.<sup>46</sup> Though both senses capture what it means to occur at an instant, they entail radically different philosophical commitments. Thus Avicenna writes,

We say that the nonexistent or existent happening ‘all at once’, in the sense of occurring in a single instant (*ān*), is not necessarily the opposite of what either gradually (*qalīlan qalīlan*) ceases to be or comes to be; rather, it is more specific than that opposite. [The opposite of what comes to be gradually] is what does *not* proceed gradually to existence or nonexistence. ... This holds true of (1) what occurs all at once; but it also holds true of (2a) the thing which *does not exist* during all of a certain time, but does exist at [time’s] limit (*ṭaraf*), which is not time, or (2b) the thing which *does exist* during all of a certain time, but does not exist at [time’s] limit, which is not a time. For indeed it is not the case that these [last] two exist or not exist gradually ...<sup>47</sup>

Avicenna clarifies the distinction between considering an instant as an extremity (*nihāya*) of a period of time rather than a limit (*ṭaraf*).<sup>48</sup> Thus, according to Avicenna, if one takes ‘instant’ to mean an extremity of time, one treats the instant like an endpoint or termination of a period of time that severs time’s continuity. In that case, though, just as one period of time, delimited by its extremities, immediately follows another period of time, delimited by its extremities, so one instant would immediately follow and so be adjacent to another instant. Inasmuch as the instant, in this view, breaks the continuity of time such that the two periods of time are not continuous, so must each period of time have its own distinct extremities that are immediately adjacent to one another. The overall effect of treating the instant as an extremity, according to Avicenna, is a type of temporal atomism, which he had precluded.

In contrast, if one takes ‘instant’ to mean time’s limit, then no two instants are ever immediately adjacent to one another as can happen with periods of time. Rather, the instant is like a point for which one can posit other points indefinitely close to it, but never a point that is immediately adjacent to it. Thus, to formulate Avicenna’s suggestion (admittedly somewhat anachronistically) in modern mathematical terms, if one takes some instant  $t$  as a limit, then for any other instant  $t'$ , no matter how close one wants to take  $t'$  to  $t$ , then there is another instant  $t''$  that is not identical with  $t$ , but is closer to  $t$  than  $t'$ . Since this same analysis holds true of  $t''$ ,  $t'''$  and so on, one can get indefinitely close to  $t$  without being immediately adjacent to it.

Given this interpretation of Avicenna’s text, it can be suggested that his notion of a limit is closely akin to a modern mathematical notion of limit. Obviously he does not have the advantage of appealing to the subsequently developed mathematical machinery of Cauchy, Dedekind, Weirstrass or Reimann to explain his account of a limit. Indeed it is not clear that Avicenna would have wholly welcomed a purely mathematical analysis of his notion of a limit. For Avicenna, mathematical analysis and

<sup>46</sup> See Jon McGinnis, ‘Ibn Sīnā on the now: text and commentary’, *American Catholic Philosophical Quarterly* (1999), 73, 73–106 for a discussion of Avicenna’s notion of an instant (Arb. *al-āna*, literally ‘the now’).

<sup>47</sup> *Aṭ-Ṭabī‘iyāt*, II.12, 161, 14–18; added emphasis.

<sup>48</sup> For a more detailed discussion of the following point see Jon McGinnis, op. cit. (46), 92; and *idem*, op. cit. (40), 19–21.

the objects of mathematics involve mental constructs abstracted from concrete physical objects, where the physical objects themselves set the constraints on the constructions that the mathematician can make.<sup>49</sup> In one very important respect, then, the science of mathematics is subordinate to the science of physics. Mathematical objects, which for Avicenna would include our modern notion of a limit, are derived by abstraction from physical objects, from composites of form and matter. As such, physics is ontologically and explanatorily prior to mathematics, thus by Avicenna's lights to use mathematics to explain and provide the basis for a notion of the limit would be to put the proverbial cart before the horse. Consequently Avicenna in a certain respect is hindered by the necessity of phrasing his conception of a limit in Aristotelian language and within an Aristotelian physical system that is at odds with the concept that he wants to convey. Nonetheless, within the strictures of the natural philosophical system in which he is working, he is certainly moving towards our modern notion of a limit.

This notion of a limit is what underlies Avicenna's account of motion's first perfection or actuality, motion as an intermediate limiting point in which there is neither a before nor an after. One considers the motion of an object over smaller and smaller spatial and temporal intervals. As one considers indefinitely smaller and smaller intervals across which and during which the object moves, the object's motion approaches an intermediate point as its limit, and at that limit there is indeed motion at an instant. Thus Avicenna has the rudiments to make a philosophically respectable account of motion at an instant, and consequently to make sense of motion as a univocal term subsuming the various kinds of motion. Indeed, Avicenna integrates his idea of motion at an instant thoroughly in his physics. Not only does he use it to define the form of motion, but also to explain (the moment of) substantial change.<sup>50</sup> It is again assumed as a premise in his argument against the possibility of circular motion in a void.<sup>51</sup> It is likewise central to his account of the changing present.<sup>52</sup>

I conclude with a historical observation about Avicenna's notion of a limit, particularly with respect to the Aristotelian sources appropriated by Avicenna in developing his notion of a limit. Avicenna's conception of a limit is not framed in mathematical terms, but in the language of Aristotle's physics. It should come as no surprise that at least some of the elements that Avicenna uses in his formulation of a limit are already present in Aristotle, most notably Avicenna's appeal to a continuous magnitude's being potentially divisible infinitely in his description of a limit.<sup>53</sup>

Although Aristotle is quite adamant that an actual infinite is impossible, he does allow for a potential infinite.<sup>54</sup> Something can be potentially infinite either by addition (*prothesei*) or division (*diaresei*). When something is potentially infinite by division, one successively takes away portions of some magnitude at a fixed ratio. For example,

49 See Ali A. al-Daffa and John Stroyles, 'Ibn Sīnā as a mathematician', in *Ibn Sīnā, Doğumunun Bininci Yili Armağani* (ed. Aydın Sayılı), Ankara, 1984, 67–140, especially Section VI, 'Foundations'.

50 *Aṭ-Ṭabī'iyāt*, II.3, 98, 9–101, 7.

51 *Aṭ-Ṭabī'iyāt*, II.8, 127, 1–18.

52 *Aṭ-Ṭabī'iyāt*, II.12, 161, 12–162, 14.

53 For a discussion of how Avicenna conceives the infinite and how he understands Aristotle's theory of the limit see his *Letter to the Vizier Abū Sa'd*, op. cit. (43), 'On the reality of the infinite', particularly 29–36.

54 *Physics* III 6.

one divides some magnitude by half and then half of that and so on. So one might take half of a distance, then one-fourth, then one-eighth and so on, getting progressively smaller increments. Clearly, in the case of continuous magnitudes such a process can go on indefinitely, giving one ever smaller magnitudes. In his description of a limit Avicenna has obviously completely taken over Aristotle's notion of a potential infinite. There is therefore one sense in which Aristotle and Avicenna had the same understanding of a potential infinite with respect to division. Yet in another sense they did not understand the same thing. To use an idea from N. R. Hanson, Aristotle and Avicenna saw the result of a potentially infinite division *as* different things, rather as two people viewing the famous duck-rabbit drawing are looking at the same line drawing and yet one sees it *as* a duck, while the other sees it *as* a rabbit.<sup>55</sup> Aristotle saw the result of a potentially infinite division *as* something always possessing a certain magnitude, while Avicenna saw it *as* a limit. Moreover, since Aristotle, and Albertus for that matter, saw this resultant as a certain small magnitude, he saw that motion would always occur over some spatial magnitude and so require a period of time. In contrast, since Avicenna saw the result of a potentially infinite division as a limit, he saw that there could be motion at a point, namely at a limit, and so at an instant. Subsequent history showed that it is the concept of motion at a limit, anticipated by Avicenna, that has had the greatest applicability to problems in physics, even if the physical problems with which Avicenna was concerned are not the same problems with which contemporary physicists are concerned.

The characterization of motion that Albertus was so quick to dismiss, the thought that motion is a univocal term identifying a generic form of motion, may very well have been, and I believe was, the one that Avicenna adopted. Albertus's misclassification of Avicenna's view of motion has in turn hindered modern historians of science and philosophy from fully appreciating Avicenna's own analysis of motion's form. Indeed, in Avicenna's development and articulation of what such a form must be like, he may have helped pave the way to, or at the very least anticipated, the modern notion of a limit. Here we may well have a chapter in the history of the calculus that has yet to be written.

<sup>55</sup> Norwood R. Hanson, *Perception and Discovery: An Introduction to Scientific Inquiry*, San Francisco, 1969, Part II.