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## Body Motion and Graphing

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We analyze how 2 students used a computer-based motion detector in the context of individual interviews. Although 1 student's work is exemplified through transcript and commentary, the themes and discussions have evolved from our study of both students' work with the motion detector as they interacted with the interviewer. Our analysis reveals 3 themes: tool perspectives, fusion, and graphical spaces. Both students developed tool perspectives that enabled them to plan how to move so that they could create and interpret graphs by kinesthetic actions. The theme of fusion explores their emergent ways of talking, acting, and gesturing that do not distinguish between symbols and referents. Graphical spaces reflect our account of episodes in which a change in how they used the motion detector prompted them to investigate the tool anew. Our conclusions contribute to a reconceptualization of the nature of symbolizing, the learning of graphing, and the links between children's and scientists' graphing.

Our analysis of how children use a computer-based motion detector to make sense of symbolic expressions in the form of Cartesian graphs of position versus time reveals three themes: tool perspectives, fusion, and graphical spaces. We exemplify our analysis with excerpts from individual interviews with Eleanor (age 10) and Dina (age 9), in their initial work with a motion detector.<sup>1</sup>

Through tool perspectives, we look at Eleanor's and Dina's efforts to make sense of the tool's graphical responses to body motion. Through fusion, we explore Eleanor's and Dina's developing ways of talking, acting, and gesturing that merge symbols and referents: visual traits of the graph with qualities of their own motions. Through graphical spaces, we reflect on our account of interview episodes in which a change in the use of the motion detector prompted Eleanor and Dina to temporarily ignore what they understood about the tools and investigate it anew.

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<sup>1</sup>The motion detector we used was provided by Lipman Co. (Israel).

Our analysis of these learning episodes strives to grapple with two interrelated questions: First, what emerging resources do students bring to make sense of graphing? When we talk about “resources,” we do not have in mind isolated rules or memorized examples but rather experiential domains, that is, bodies of life experience that the student brings to the situation full of intuitions, expectations, and ways of talking. These are emerging resources; they are not just “retrieved” as if they were modular, ready-to-use objects but are continuously changed and transformed by the local history of the student’s situation. Second, what aspects of the situation are significant to the student, and how do they shift? Aspects that Eleanor and Dina at first perceived as irrelevant, such as graphs appearing on the screen from left to right, at times shifted to the foreground of their inquiries, whereas other aspects, such as keeping within the range of the sensor, became background knowledge.

### DESCRIPTION OF THE STUDY

This study consists of open-ended mathematical conversations (i.e., interviews), in which the participants—the interviewer and the student together—investigate ways of using a motion detector and graphs. We participate in these conversations to gain insights not only into how mathematical situations appear to the students but also into our own mathematical understandings (Confrey, 1992). By striving to relate Eleanor’s and Dina’s shifting perspectives with our own sense of the nature of graphing, we come to conclusions that broaden our conceptions of symbolization, the learning of graphing, and the links between children’s and scientists’ graphing.

The motion detector used in this study consists of a small object or “button,” a sensor or “tower,” and a computer. The child holds the button within “view” of the sensor, or she places it on a moving object, such as an electric train. For the interviews reported here, we have set the interface so that the computer monitor displays in real time a graph of the changing distance between the button and the sensor tower, that is, a graph of distance versus time (see Figure 1). Beyond a distance of 3 m, the detector does not measure, and the computer graph flattens. We set the graphical scales from 0 to 18 sec and from 0 to 400 cm. The computer screen displays a grid of dotted lines marking seconds and meters (see Figure 2).

As the child uses the motion detector during the interviews, the goal of the interviewer is to understand the child’s thinking and learning. Although we plan a possible sequence of tasks for the interviews, we deliberately leave them open-ended. Each interview is a unique event; we do not know ahead of time what will be rich. We ask the child to talk aloud about what she is thinking. The interviewer responds naturally with expressions of interest and also intervenes at times with restatements and questions to focus the child’s attention on aspects of the situation or on the child’s own statements or actions.<sup>2</sup> We videotape the interviews and meet

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<sup>2</sup>We build on the notion that the interviewer cannot learn what the child thinks without affecting how the child is thinking.

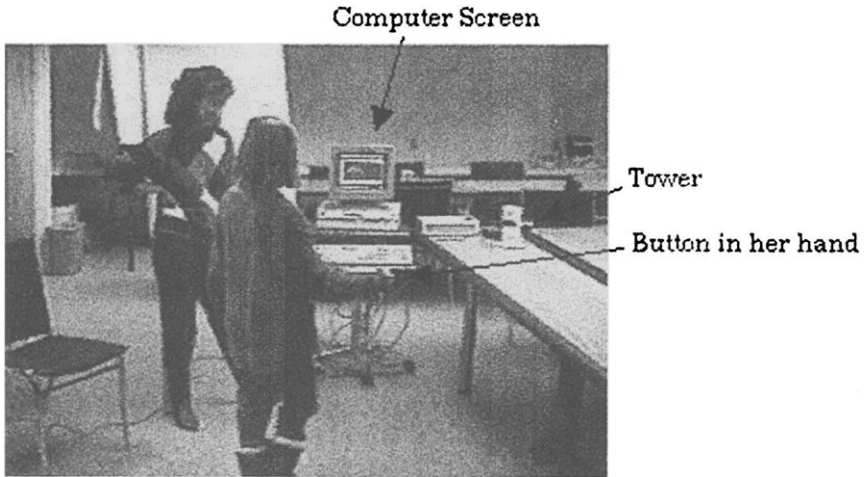


FIGURE 1 Eleanor and Tracey using the motion detector.

as a group to discuss them and to plan the next interview. The questions we pursue when we analyze the videotapes often arise from the data we observe. We do not limit our interest to what the child knows; we also look at what interests her, how she goes about finding out more, and how she relates to the interviewer.

For this study, we have interviewed 12 children individually, ranging in age from 7 to 14 years old, and focused on their ideas about motion and graphing. Each child was interviewed approximately four times in sessions usually lasting 1 hr. We decided to analyze in depth Eleanor's and Dina's initial experiences with the motion detector because their sessions helped us learn and clarify ideas that became useful for our analysis of the interviews with all the children. Although this article reflects the analysis of the work of both girls, examples will come primarily from one student's (Eleanor's) interviews because of space constraints. Having to choose between Eleanor's or Dina's transcriptions, we decided to preserve the full excerpts with Eleanor because they are easier to follow. Certain issues, however, will be illustrated with or expanded on by using excerpts from Dina's work.<sup>3</sup>

Dina and Eleanor first used the motion detector during their second session with the interviewer, Tracey Wright. In the first session, they had invented representations without words to instruct Tracey to move a handheld toy truck and an electric train in particular ways that each one established. Each child in her separate interview came to include instructions whether to go fast or slow, where to start or stop, and when to change direction.

<sup>3</sup>For an earlier draft that includes in-depth analysis of Dina's initial use of the motion detector, send a check or money order for \$5.00, payable to TERC, to TERC Communications, 2067 Massachusetts Avenue, Cambridge, MA 02140.

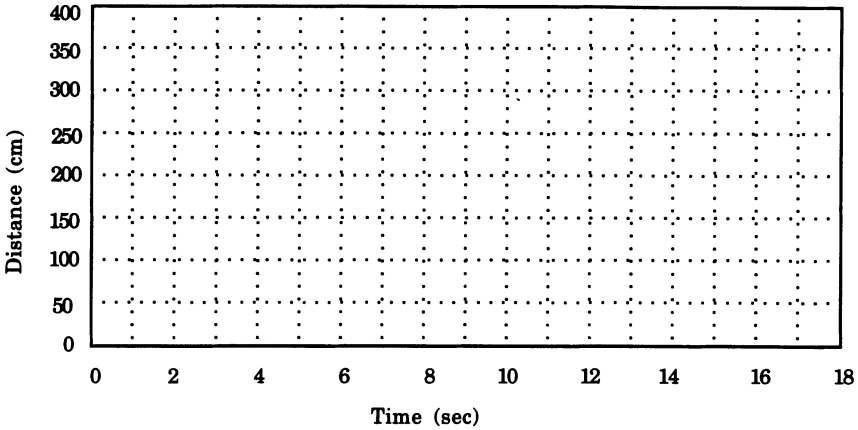


FIGURE 2 Grid displayed on motion detector.

In the second session, the children began working with the motion detector. At first, the interviewer posed no particular tasks to Eleanor and Dina but encouraged them to take charge of the investigation. Both children began by investigating what motions elicited a response from the motion detector. Each, in her own way, came to see that the line on the screen becomes higher when one moves the button away from the tower and lower when one moves the button toward it. They then made use of their findings to explore something of interest to them. Later, Tracey suggested a change of context in the use of the tool, including using two handheld buttons instead of one to generate two graphs simultaneously and attaching one button to an electric train.

With only one exception, the episodes are introduced under themes of tool perspectives, fusion, and graphical spaces following their chronological order; that is, whatever is presented later took place later in the interviews. Each theme comprises an introduction, commentaries on the annotated transcript, and a discussion section. We indicate missing pieces of transcript with an ellipsis within parentheses. Almost all the missing pieces of transcript are short exchanges that refer to procedural details. Ellipses without parentheses indicate a pause.

### CONTEXTUAL REMARKS

Many studies on the nature and learning of graphing assume some fundamental dichotomies. For example, researchers have analyzed students' misconceptions (or alternative conceptions), which are to be contrasted here with the conception of the expert grapher (Clement, 1989; Janvier, 1978). Opposing "misconceptions" to the socially accepted conception of the expert is a way of dealing with the general dichotomy between the social and the individual; the individual is seen to become "encultured" by replacing his or her idiosyncratic approaches with the socially

sanctioned approach. Others focused on the polarity between pointwise and shape-based understanding of graphs as an instance of the more general dialectic between process and object that, according to several authors, underlies all mathematical learning (Dubinsky & Harel, 1992; Sfard, 1991). Other studies have focused on the opposition between external and internal representations in which external representations, in the form of tables, graphs, or equations, are viewed as corresponding to internal representations.

By focusing on one or another dichotomy, students' performance is analyzed in reference to a standard and expert method that is well known to the researcher. Consequently, it is not surprising that the literature emphasizes what the student lacks. In consonance with an emerging research perspective (Cobb & Bauersfeld, 1995; Confrey, 1991; Smith, diSessa, & Roschelle, 1993–1994), we avoid seeing students by their mistakes to be overcome. We look instead at students' conversations, their emerging resources, and how they relate to the use of tools.

What we find questionable is *not* the use of opposed qualities or the making of distinctions but rather the acceptance of them as essential dualisms underlying thinking and learning. Our perspective is influenced by writings that are characterized by an effort to encompass at once all the aspects of ongoing human existence. We do not acknowledge classic dichotomies, such as subject–object, signifier–signified, mind–body, or internal–external, as the constitutive elements of “reality.”<sup>4</sup> Those distinctions are meaningful ways of talking about and reflecting on past events, but they are not at the foundation of the human experience. When, for example, one participates in a conversation, one does not distinguish a gesture as belonging to the body or to the mind. When one reads a text, one does not split the signifier (the letters on the text) from the signified (the story told by the text). Actually, we sense these kinds of splits as a disruption in the flow of experience. This philosophical tradition we are following argues that the meaning of symbols is to be found neither in the specific thoughts that they express nor in the objects to which they refer but in their use, that is, in the practices they serve.

Recently, a number of studies have reported a renewed exploration of the learning of graphing (Béguin et al., 1995; Boyd & Rubin, 1996; Carraher, Schliemann, & Nemirovsky, 1995; Chazan & Bethell, 1994; diSessa, Hammer, Sherin, & Kolpakowski, 1991; Kaput & Roschelle, 1995; Monk & Nemirovsky, 1994; Moschkovich, 1996; Nemirovsky, 1994, 1996; Pratt, 1995; Stroup, 1994; Tierney & Nemirovsky, 1995; Vitale, 1995; Yerushalmy, 1996). The overall view that emerges from these studies is, in our view, one in which (a) learning graphing entails the enrichment of a broad range of experiential domains, involving the refinement of visual, kinesthetic, and narrative resources; and (b) adopting of mathematical conventions does not diminish the open-endedness involved in their use. No two

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<sup>4</sup>In our opinion, landmark works from this literature are Heidegger (1927/1962), Merleau-Ponty (1945/1989), and Wittgenstein (1953).

individuals' graphing processes are identical, and no two bring the same resources to a new situation. Instead of the uniform "internalization" of the graphical conventions described in textbooks, we witness the creative, conflicting, diverse, and ever-changing ways in which students strive to make sense of graphs and express themselves using graphs.

The notion that thinking is an "embodied" activity—that the human body as a whole, not just the brain, is involved in how we conceptualize situations—has been convincingly developed in the writing of Merleau-Ponty (1945/1989) and the works of Johnson (1987) and Lakoff (1987). The bodily basis of understanding is central to this research, but in contrast with Johnson and Lakoff, our intent is not to identify core cognitive structures but to gain insight into how situations involving body motion and symbols look to particular beginning students of graphing, Eleanor and Dina.

To talk about the ways in which people construct and interpret symbolic expressions, researchers (Lave & Wenger, 1991; Mason, 1987; Meira, in press) invoke metaphors involving the act of "seeing"—such as "seeing through," "transparency," and "fields of visibility." An example of this in graph making is a situation in which a person traces a line on paper and says: "this straight line." The idea that this person expects to share requires some degree of not seeing the trace, for no actual drawn trace is a straight line. On the other hand, the trace makes pointable a field of linear relations that are the actual subject of the speaker's utterances and gestures. We are investigating how this kind of shared understanding grows, including episodes in which it does not occur, such as when the interviewer draws a curve intending to express the idea of an increase that rises continuously and the child strives to make sense of the interviewer's drawing by focusing on its unintended pictorial qualities, for instance, some pieces that look straighter than others or one end that seems vertical.

The conception of symbol use as a complex seeing highlights another question: What does come into view? Latour (1986) contributed an important analysis of this question. He described how symbols are used by people and cultures to make present the absent. A conversation that uses a map, for instance, merges the here and now of the speakers with selected features of a distant land. The represented land is brought to the site of the conversation in ways that make it partially pointable and accessible. Latour talks of these as the constitution of a "common place" that is shared by symbol users. The common place is populated by absent experiences, objects, and events that come into view within the localized here and now of the symbol users. The idea of a common place is important to our research because we explore how children make sense of symbolic expressions—Cartesian graphs representing their body motion—as a creation of a common place, which we call *graphical space*: a space that is gradually enriched by experiences and that becomes an explicit part of conversations and gestures.

The three themes presented here are interrelated in that each one is an exploration of certain aspects of a graphical space. Tool perspectives look at development of

graphical space through simultaneously exploring the qualities of the tool and the relation between actions and symbols. Fusion is about the blending of action and symbol in discourse within the graphical space. Graphical spaces explore how such spaces change as experienced by their users.

## TOOL PERSPECTIVES

As part of our everyday use of tools, we often experience a shift between the tool as an implicit aspect of our goal-oriented activities (e.g., using a knife to cut vegetables as one is making a salad) and the tool as an object of reflection (e.g., trying to figure out what type of knife is best to cut vegetables). These familiar experiential shifts have become an important theme for 20th-century philosophy. As in the famous example of the blind man's cane, cited by Bateson (1972), Merleau-Ponty (1945/1989), Wittgenstein (1953), and others, the fluent use of a tool appears to involve its transparency in the service of accomplishing goals as well as its examination when, for example, the tool does not seem to perform as expected. If this is a suitable characterization of expertise in the use of a tool, what happens when the tool is unknown and unfamiliar to the user? How does expertise develop? This article is a case study on these questions. In this instance, the tool is a motion detector, and the users are Eleanor and Dina as they encountered the motion detector for the first time.

Vygotsky's (1978) description of the development of expertise in tool use as a process of internalization, through which the individual becomes enculturated, is well known. We avoid this language, however, to preclude anchoring ideas on the external-internal dualism and to leave open the question of when and how fluency in the use of tools involves enculturation. Our analysis highlights the development of what we call a *tool perspective*. Adopting a tool perspective involves emulating the tool's sensitivity to certain aspects of motion and not to others, ascertaining conditions under which the tool is useful, and recognizing patterns of significance in the tool's products.

Interpreting X-ray images—an example of adopting a tool perspective—involves not only knowing anatomy, pathology, and so forth but also having an awareness of what is visible and invisible to the device and how idiosyncratic limitations of the tool produce visual traits to be ignored. In showing an X-ray photograph, the physician may draw attention to a little spot on the image that otherwise might be absolutely unremarkable to another viewer, whereas an optical artifact of the X-ray device that is irrelevant to the physician may look important to this viewer.

The motion detector has been designed assuming that the user would conform to certain performance requirements and limitations, such as always facing the button toward the tower and keeping the button within range of the sensor. In

addition, the designers of the motion detector adopted as “natural” certain conventional expectations, for example, that temporal graphs are always drawn from left to right or that connected lines are supposed to be seen as continuous, even though the pixel structure of the computer screen may portray them as bumpy.

When children begin to use the motion detector, they are not aware of its limitations on the kinds of actions that it can sense, and they may not share cultural expectations as they are reflected in the tool (e.g., that time cannot reverse direction). Thus, we talk about developing a tool perspective and not the tool perspective. Through their initial use of the motion detector, Eleanor and Dina began to discriminate between what *for each one of them* were idiosyncratic aspects of the tool to be ignored or just complied with and what was significant to interpret and work with to make their intended graphs. It is not the case that every aspect of tool use is either idiosyncratic or significant in a general sense. For the tool user, aspects may become issues that are not necessarily the ones intended by the designer of the tool. For example, blocking the button with one’s hand to create vertical lines was significant for some students we interviewed, although this is not intended to be so by the designer of the tool.

Tool perspectives may vary for different individuals and also change for the same individual as she experiments and reflects on the use of the actual tool over time. Developing tool perspectives enabled students to anticipate how the graph *should* look, so that if the graph did not conform to their expectations, they would try to recognize accidental mistakes in their motions or inherent limitations of the tool to explain the discrepancy.

The episodes in this theme describe the first 10 min of Eleanor’s use of the motion detector. Our central aim is to illustrate specific aspects in Eleanor’s process of developing a tool perspective for the motion detector, hoping that such an account might broaden our understanding of what is involved in becoming a fluent tool user. Our analysis strives to articulate three contributions:

1. As they held the button, the foremost quality of the tool was, for Eleanor and for the other students, its responsiveness to their body motion. They began by striving to identify which elements of their body motion the motion detector sensed. Eleanor’s engagement in this process is the focus of the first section, *Responsiveness of the Tool*.

2. We see how Eleanor gradually envisioned actions from a tool’s point of view, distinguishing which actions were noticeable and which were indiscernible. She endeavored to find out how what she perceived as significant and idiosyncratic traits in the behavior of the tool affected “its” perspective. Eleanor’s activity illustrates this subject in the second section, *The Tool as a Point of View*.

3. Finally, the third section, *The Emergence of Logical Necessity*, illustrates how Eleanor considered what types of graphical productions are possible, impossible, or difficult, expressing her growing sense of how the tool assumes a certain logic.

Keep in mind that these three foci of analysis do not attempt to separate distinct sequential phases. Eleanor's view of the responsiveness of the motion detector to her body motions, for example, does not end with the episode included in the first section. Rather, we have chosen to highlight a certain aspect in our analysis of each set of episodes—even though it develops throughout the whole interview—because of its conspicuousness at that particular time.

Before starting the analysis of the episodes, we need to clarify what we mean by *actions* and *responses* because we make repeated use of these terms. By actions we mean situated and intentional attempts to affect one's circumstances (Suchman, 1987). Actions are situated because they incorporate, on an ongoing basis, local and contingent aspects of the actor's setting. Actions are intentional because they express desires to accomplish shifting and, at times, conflicting purposes that emerge from the history of the situation itself. We think—as in Ryle's (1963) famous discussion on the difference between winking and closing one's eye as a result of a nervous tic—that understanding actions involves grasping the lived situations, intentions, and histories surrounding their occurrence.

Schön (1987) and Bamberger and Schön (1991), through their description of the design process as a “conversation with the elements of the situation,” suggested that actions and responses constitute each other continuously. Responses often bring to the fore unexpected issues and questions. For example, a man hammering a nail on the wall hits the nail expecting that it will penetrate the wall a bit more, but instead, the nail bends; to him, this is an unintended response. He then needs to cope with the unintended by, for example, starting all over with a new nail. Some responses prompt the actor to adopt a more reflective attitude. For example, the nail might, instead of going deeper into the wall, stay at the same depth and produce a metallic noise. The man would then wonder what the nail could be hitting and whether another location on the wall would be more suitable. Responses emerge within the actor's interpretive activity; moreover, actors are often trying to figure out whether a certain event counts as a response or not.

The episodes described here took place in the second session with Eleanor during the initial 10 min of her use with the motion detector. The three sections include episodes according to the time scale shown in Figure 3.

## Responsiveness of the Tool

Tracey introduced the motion detector, highlighting the button, the tower, and their mutual relation<sup>5</sup>:

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<sup>5</sup>This was Tracey's first interview using the motion detector.

Minutes counting from the moment in which Eleanor started to use the motion detector.

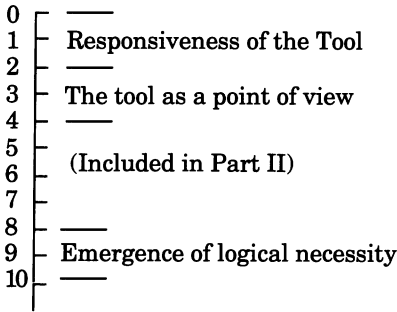


FIGURE 3 Time scale of minutes from the moment at which Eleanor started to use the motion detector.

Tracey: This is a button, and this is a tower. And this is how this works. If I push F1 it'll start [pushes F1], and you can move that [indicating the button in Eleanor's hand]. And it's going to respond to the tower. So [a line begins to appear on the computer screen, Segment 1; see Figure 4].

Eleanor begins her use of the motion detector to investigate the graphical responsiveness of the tool to her kinesthetic actions (see Figure 5).

Eleanor: [As she moves her hand sideways the line lowers a little bit, Segment 2.] Let me move it farther away. [She walks backward away from the sensor; the line rises in Segment 3; see Figure 4.] The line goes up. [She continues walking backward and the line flattens, Segment 4.] Maybe this is the farthest it can go. What if I move it up higher? [She walks forward and moves her hand up high, Segment 5. Then she walks backward a little and forward all the way to the sensor, Segment 6.] The closer to the tower it gets, the lower, I think. [She walks backward away from the sensor, producing Segment 7.] And I think it gets. It gets higher until this line. [She points to the screen where the line has flattened in Segment 8.] And then it [the graph reaches the end of the scale and stops]. Wait, oh, I can't do it anymore. How do you clear it [the computer screen]?

Eleanor's initial sideways movement of her hand did not engender a significant response on the computer screen (Segment 2). As soon as she tried a backward movement with her hand, however, the tool seemed to "wake up" (first curvy piece in Segment 3). This graphical response encouraged Eleanor to further pursue the

backward movement by walking steadily away from the tower. Beyond a certain distance, the line flattened even though she continued to walk back (Segment 4). The silence of the tool, as Eleanor stepped outside an unmarked region, made her aware of apparent boundaries and inspired her comment: "Maybe this is the farthest it can go." The "it" of her comment was probably not the button she was holding in her hand—there were no limitations for the location of the button—but the

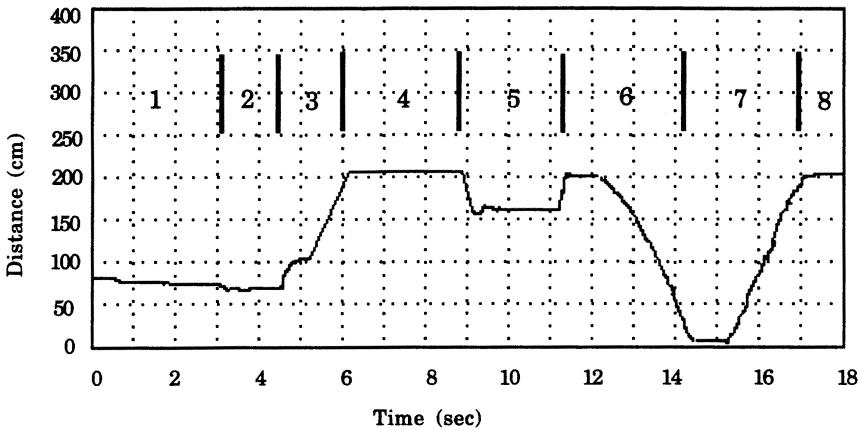


FIGURE 4 Eleanor tries different types of arm movements to create her first graph.

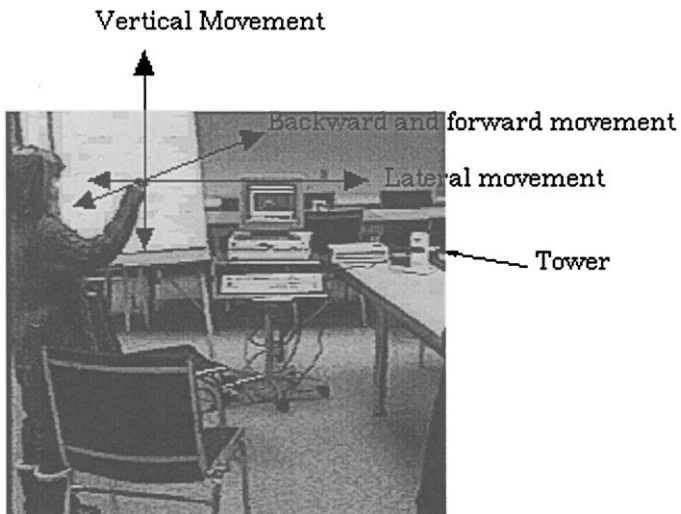


FIGURE 5 Eleanor: "Let me move it farther away."

engagement between the button and the tower. Striving to break the passiveness of the tool, Eleanor lifted the button up, perhaps expecting the graph to go up, at the same time that she walked a bit closer. Eleanor ascribed the jump down of the graph in Segment 5 not to the upward movement of her hand but to her walking forward. This change prompted her to express a key insight: “The closer to the tower it gets, the lower, I think.”

We want to highlight two interrelated aspects of Eleanor’s sense that closeness to the tower is expressed by the height of the graph: Eleanor accounted for her experience of the most remarkable features of the tool’s responsiveness by articulating actions—her own getting closer to the tower and the graph getting lower—and Eleanor envisioned a qualitative relation between the two measures, namely, that a decrease in one (getting closer) corresponds to a decrease in the other one (getting lower). More than specific correspondences (e.g., certain heights on the graph corresponding to certain distances to the tower), Eleanor pointed out a relation between changes along two continuous magnitudes. We shall see later how Eleanor began to incorporate quantitative and pointwise elements.

As Eleanor produced the graph in Figure 6, she wondered about the tool’s response to stillness.

Eleanor: OK, so let’s see. So. [Eleanor moves slowly away from the sensor, Segment 1. Then she stops partway back and holds the button still in front of her.] What if I just leave it here? Will it keep on going? [The graph is a horizontal line, Segment 2.] Yeah. What if I move it up [Eleanor lifts her arm up, Segment 3], wait, let me see. So it goes ... [she walks closer to the tower, Segment 4], the closer I go, the lower it goes.

Grasping the responsiveness of the tool involves awareness of what aspects are not responses, in other words, what the tool does on its own. Pursuing her systematic investigation, Eleanor found an example of graphical activity (going to the right) that occurs in the absence of kinesthetic actions: The graph “keep[s] on going” even if she “just leave[s] it here.” When Eleanor tried holding the button up high again, she had not ruled out the tool’s sensitivity to vertical motions. Our work with Eleanor and other children showed us that a sense of responsiveness, such as “the closer I go, the lower it goes,” does not preclude other possibilities; the tool might respond in that way and still be sensitive to other body actions that do not seem to affect closeness to the tower, such as vertical hand motion.

We have seen that, during her initial interaction with the motion detector, Eleanor tried to grapple with two issues: What aspects of one’s body motions elicit responses from the tool? What does the tool do on its own, independent of one’s actions? Eleanor’s ensuing experimentation with the motion detector pursued these

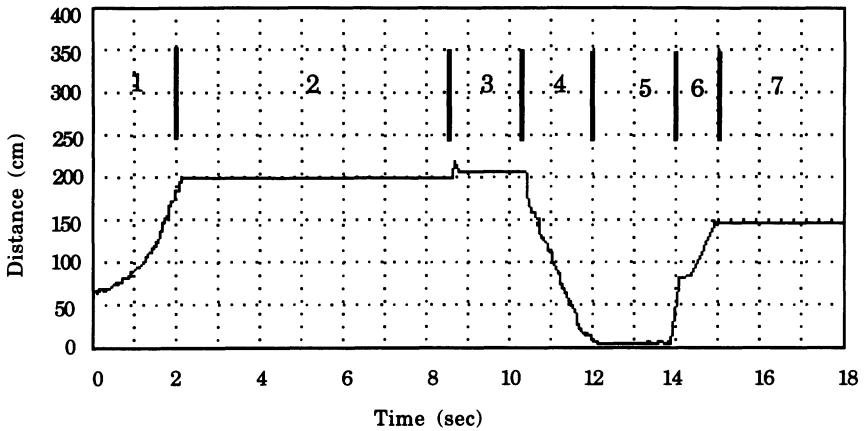


FIGURE 6 Eleanor: "What if I just leave it here?"

same questions in new ways, portraying her remarkable refinement of what is to be taken as significant or idiosyncratic.

### The Tool as a Point of View

Continuing with creating the graph in Figure 6, Eleanor tested her hypothesis that the graph would go higher as she moved the button away from the sensor.

Eleanor: But what if I go like far and like, no [she walks away from the tower, creating Segment 6; then Eleanor sees that Segment 7 is flat even though she still moved farther away from the tower]. Hmm, why isn't it going as high? Maybe I went too far.

As Eleanor walked away from the tower, the graph displayed a rising line (Segment 6); however, contrary to her expectations that the line should continue to rise, at 1.5 m the graph flattened out (Segment 7). This flattening happened because Eleanor turned the button away from the tower so that the optical link between the tower and the button did not work anymore. In producing this graph, Eleanor encountered two kinds of flatness (Segments 2 and 7). Segment 2 was the graphical response to her stillness ("What if I just leave it here? Will it keep on going?"), but Segment 7 was puzzling to her. She did not perceive Segment 7 as a response to her actions because she was not aware of doing anything distinctive when the graph went horizontal; the computer had done it *on its own*. Eleanor resorted to the idea of the maximum ("I went too far"), but this explanation could not account for

Segment 7 because Segment 2 had been even higher. The result was Eleanor's feeling that there was something strange about Segment 7 and the maximum height. To Eleanor, this reopened the issue of the maximum height reachable by the graph.

Eleanor started a new graph (not shown). She turned the button to face the tower again, and the graph became horizontal around 1.5 m. She struggled to interpret it ("oh, I know it goes from ... I think it goes from like ..."), but Tracey decided to make explicit the issue of the button orientation.

Tracey: I just noticed one problem, and I think it has to do with the way you're holding it [the button]. If you hold it [the button] like this [toward the tower], it [the tower] can see better than if you turn it [the button] that way [away from the tower], it [the tower] can't see what you're doing. [Eleanor continues to generate the graph, and Tracey notices that she had accidentally turned the button away from the tower again.] You're turning it away again.

Tracey's explanation made Eleanor aware that the orientation of the button is a factor to consider in how to use the motion detector. Holding the button toward the tower became an aspect of good performance with the tool, an action to conform with, rather than to play with. This is a central trait of the idiosyncratic aspects of tool use. To eliminate turning the button as a component in the graphical production or interpretation, Tracey talked about the button's orientation in the context of the tower seeing the button, an image that Eleanor would use a little later.

Eleanor continued to investigate what happened when she walked far away as she created a new graph (see Figure 7).

Eleanor: (...) Let's see. What happens when you're really far away? (...) [Graph begins. She walks back as far as she can in the room. The line flattens in Segment 2 before she gets all the way back.] I think I'm going to leave it like this for a minute. [She then reaches further back, holding the button as far away from the tower as possible.] Hmmm, I wonder if I hold it up high? [She reaches high up in the air while she stands way back, then she lowers the button to shoulder height, and, producing Segment 3, walks in then slightly back. After she has completed the graph she reflects on it.] No. I think that the ... this is the ... maybe this [points to the horizontal line, Segment 2] is the highest it can go. I don't know. But, um, so if you get closer to it [the tower], it [the graph] gets ... it goes down low [points to the low part of Segment 1]. And if you get far away it goes up, well, when you're walking farther away it goes up high [traces the upward piece in Segment 1]. Then it ... then it goes to about here [Segment 2]. And then, but I don't think it really makes any difference if you go like

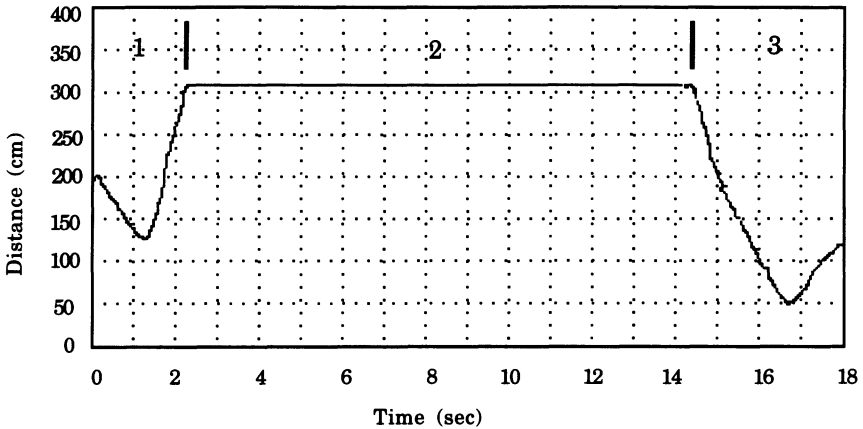


FIGURE 7 Eleanor: "Let's see. What happens when you're really far away?"

that [bending knees and moving button by reaching down low and moving it up high].

The flatness of Segment 2 was the key aspect that Eleanor experimented with. As Segment 2 appeared on the screen, she tried different actions to break its uniform horizontality (staying still, reaching as far back as possible, and reaching high up). The computer's failure to respond to her actions lead her to conjecture "but I don't think it really makes any difference if you go like that."

Tracey: It [the up and down movement] doesn't make a difference?

Eleanor: Well, maybe it does. I don't know. I'll try it. [Eleanor begins another graph, not pictured. She holds the button down on the floor and notices that the table might block the tower from seeing the button]. Maybe it [the tower] can't see [the button]. Wait, can I hold this tower like [on the floor], or should I just leave it there [on the table]? Because it [the tower] might not be able to see it [the button], right? If I put it [the button] down here [under the table] (...)

Her subsequent use of the motion detector was mostly within the space visible to the tower, preserving the button's orientation so that it could be sensed by the tower.

In this episode, Eleanor incorporated idiosyncratic aspects of the tool into her performance. Eleanor learned to recognize and enact peculiar conditions (not too far from the tower, the button always visible to the tower, etc.) within which the meaning of graphs is to be discussed and envisioned. It is from this context that we talk about Eleanor's envisioning the tool as a "point of view." Eleanor learned to

recognize aspects of her body motion that were visible to the motion detector and conditions to be preserved so that the tool remained sensitive to them.

### The Emergence of Logical Necessity

We have reviewed how Eleanor came to perceive the graphical height on the computer screen as expressing movements toward and away from the tower. This allowed Eleanor to imagine sequences of actions to achieve a certain graph. When an actual graph did not validate her expectation, Eleanor sought to discover whether it was an issue of idiosyncratic tool performance or her misunderstanding of what the graph meant by actions.

We examine here how Eleanor and Dina thought about graphs that were possible, impossible, and difficult.

*Eleanor.* The following episode with Eleanor took place after 8 min of experimentation with the motion detector.<sup>6</sup> Having just created the graph in Figure 8, Eleanor wondered aloud about the possibility of creating a vertical line. Throughout this episode, “it” refers to the graph line.

Eleanor: I wonder if you could get it to go straight up, not like diagonal? Probably you couldn’t because if it would go straight up it would have to just be the same time, because it’s moving along [tracing a horizontal line from left to right across the computer screen], no matter what you do.

Tracey: Moving along in time?

Eleanor: Yeah, so you’d have to kind of stop the time and go like that [gesturing a vertical line on the computer screen]. And go like this [moving as if to rush away from the tower, still holding the button]. Because, because it’s moving along this way [to the right on the screen] the same time it’s going that way [straight up] (...)

Eleanor expressed the impossibility of the vertical line (“Probably you couldn’t”) based on the notion that the graph moves to the right “no matter what you do.” Eleanor recognized moving to the right as a significant aspect of the tool: It is not a matter of limitation, performance, or inaccuracy but rather a crucial element in imagining the creation of graphs.

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<sup>6</sup>See the Fusion section for a detailed description of Minutes 4 to 8.

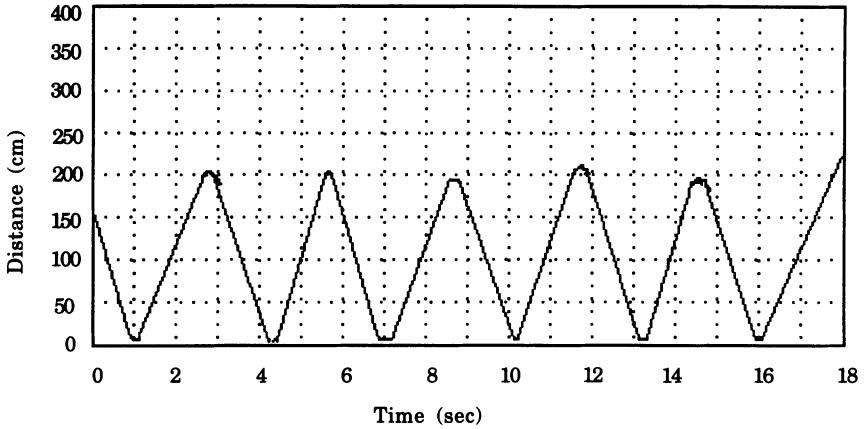


FIGURE 8 Eleanor: "I wonder if you could get it to go straight up, not like diagonal?"

Eleanor was developing a perspective from which she could emulate the tool's view of motion, space, and time. The next exchange expresses Eleanor's growing sense that the vertical line is not just an isolated case but one extreme along a continuum of possibilities.

Tracey: Do you think you could make a steeper line than this [referring to one of the slanted lines in Figure 8]? Maybe you can't make it go straight up, but maybe you can make it a little bit ...

Eleanor: Maybe, maybe if you do it faster.

Tracey: OK, shall we try that?

Eleanor: [See Figure 9; she starts a new graph, running a short distance toward the tower and back then stands still moving her arm quickly forward and back.] (...) That's almost straight up [referring to a line or lines that are nearly vertical in Figure 9].

Tracey: Yeah, it is almost straight.

Eleanor: So you can make it kinda go like this [holds finger slightly slanted to the left] even though you can't make it go like straight up [holding finger vertically]. You can get pretty close if you do it faster. And when you do it slower you can get like [gestures a line close to horizontal] slower.

Eleanor incorporated into her tool perspective a way of showing speed: by closeness to the vertical. In doing so, she enriched her appreciation of the significant; it made it more likely that, if she were to see a graph that would become more horizontal when she moved more quickly, she might assume that the tool

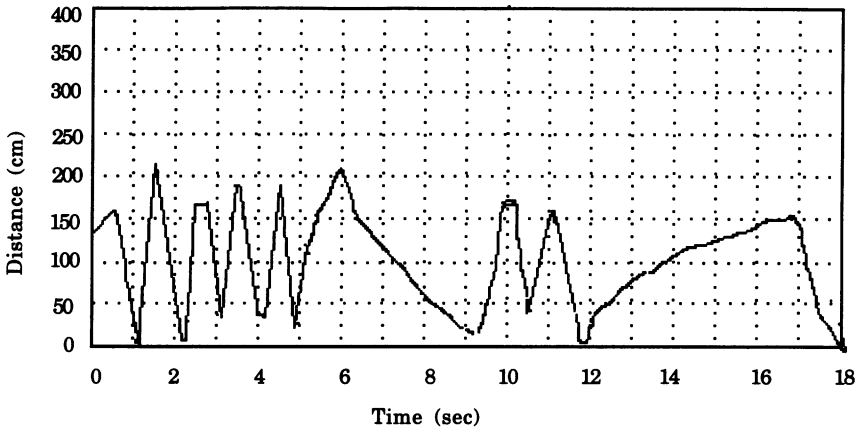


FIGURE 9 Eleanor: "That's almost straight up."

did not work properly because a sense of how it should work had become compelling to her.

*Dina.* Dina began making sense of the graphical responsiveness to her actions by interpreting the graphical height as an expression of speed. This view may have grown out of her experience of inventing representations on paper with Tracey, in which Dina used vertical height to indicate speed. She repeatedly reconstructed how she had moved the button to be consistent with this interpretive framework. Gradually, Dina began to include elements of distance to the tower as a significant aspect; her failure to produce a "high" graph with just fast arm motion, for example, led her to introduce a distinctive kinesthetic action, that is, getting farther from the tower where she remained still except for her arm motion. As Dina prepared to produce a new graph, she expressed a blending of being far and fast as well as a kinesthetic "mixing" of walking and moving her arm.

Dina: (...) OK. I'm just going to get far. I really want to get it fast. [She walks back to start far from the tower.]

Once Dina felt satisfied that she could make the graph go up and down, she began to try to draw. She soon became aware of some things that are impossible to draw with the graph. She created the graph shown in Figure 10.

Dina: (...) Well, here [pointing somewhere near the last part of the graph] it kind of came out messy, but I was trying to make an *N*. It didn't fit. I wonder if you could draw something trying that.

Tracey: That would be interesting. What would you like to try to draw?

Dina: Something easy. OK, maybe a box. That's easy. Pretty easy. Can you get this to go back? Like once it's over here [in the right-hand part of the screen] to go back [motioning with her hand toward the left on the screen]? I don't think you can, but ...

Tracey: Yeah, not the way it's set up right now.

In this episode, Dina discovered that something might be easy to draw and impossible to graph. Note how a characteristic that was present in the graphical productions from the very beginning, the movement to the right, only now became an issue for Dina. As soon as she intended to draw a square, the directionality of the graph presented itself as something to cope with. Expressing the significance of this new awareness, Dina started to think about what letters were possible and impossible to write using the motion detector.

Dina: Should I try another one?

Tracey: What other drawing could you make?

Dina: Hmm. Could I try to write something? I don't know what, but ... *the*? [Tracey: OK.] Or *at*, as long as it's easy. Except how to make an *a*, I don't know how, because it doesn't finish the circle [small *a*]. It would only be a bump.

Tracey: Yeah. Huh. So the *N* works [Dina: Yeah.], but the *a* doesn't.

Dina: A *T* I don't think would work either. It [capital *T*] would be like an upside down *L*. It wouldn't have the stick in the middle of the straight one at the top. So it can't be a *T* or an *a*.

Dina's insight about the constraint of the graph going always to the right was part of her construction of the significant because it constituted a general principle

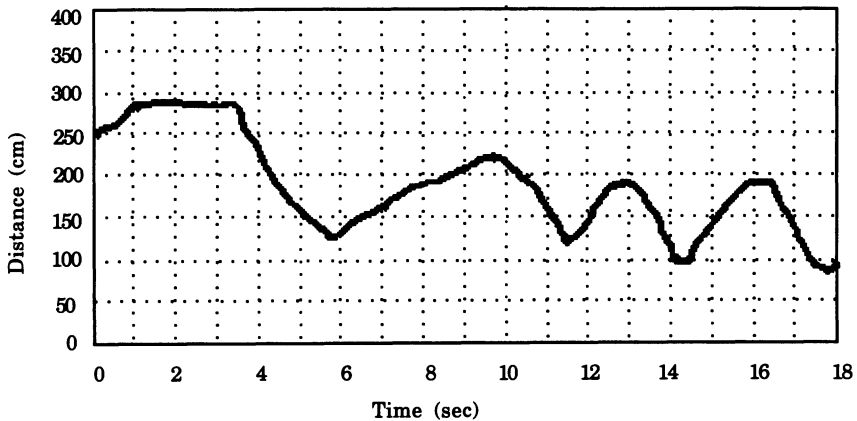


FIGURE 10 Dina: "But I was trying to make an *N*."

underlying the relation between the computer graphs and her actions. No matter what one does, the computer graph keeps moving in the same direction. On the one hand, Dina saw this behavior as an intrinsic attribute of the motion detector, part of what the computer does by itself; on the other hand, she took from it a strong sense of how she should read a graph (from left to right without changing direction) and imagine the sequence of actions that would produce it (left to right is first to last). By creating a new taxonomy of possible and impossible drawings, Dina used the directional principle to examine under a fresh light a broader domain of hand-drawn shapes; in other words, her realization went beyond the mechanics of the tool enabling her to adopt an alternative view—the view of the tool—toward familiar shapes. Note that, in this exchange, Dina analyzed possible, impossible, and difficult graphs without actually trying to produce them but by just envisioning their creation. It is in this sense that we talk about an evolving tool perspective that incorporates whatever is significant to the tool user against a background of idiosyncrasies to be complied in the actual performance with the motion detector. Her tool perspective, incorporating the logic “principle” of always moving to the right enriches her imagining of what is likely, impossible, or necessary.

Our analysis of Dina’s views on the logical necessity, making some letters possible and others impossible, strives to identify how it was lived by her. Expert graphers may recognize, for the same taxonomy, elements of cultural value that are not necessarily shared by Dina. For example, the impossibility for a temporal graph to change direction may seem to a knowledgeable adult, such as Tracey, a natural consequence of the irreversibility of time. However, later in this episode, Tracey and Dina had the following exchange.

Tracey: And can you make it [the line graph] come back [to the left]?

Dina: I don’t think you can.

Tracey: How come?

Dina: I have no idea. I would think just because the computer isn’t made to do it.

That Dina used the graph’s rightward movement repeatedly to imagine possible, difficult, and impossible graphs in the form of letters, although she attributed this trait to a designer’s choice, suggests that a competent use of the motion detector not always implies sharing the cultural background of the choices made for its design.

## Discussion

*Responsiveness of the tool.* The initial “problematic” for the students using the motion detectors (Confrey, 1991) was to find ways to provoke salient graphical responsiveness of the tool to their own actions as they held the button.

For instance, during the creation of the graph in Figure 4, Eleanor walked backward as she commented that the “the line goes up.” At a certain point, when she had not performed any distinctive motion, the graph suddenly flattened. She perceived the graph’s flattening as something that the tool did by itself. Grappling with the responsiveness of the tool involved dealing with the concord and discord between her intended graphs and the resulting graphs. Eleanor raised up her arm, intending to recover the lost responsiveness in the previous example. She noted that the vertical motion made no difference. The local circumstances—shaped by her own actions—were crucial to how Eleanor made sense of the events on the graph.

*The tool as a point of view.* The metaphor of the tool as a point of view invokes the familiar experience of trying to imagine an object from a different viewer’s position. Envisioning an object from an alternative point of view involves recognizing what aspects of the object will be significant from the new angle as well as assuming a background of idiosyncratic conditions to preserve the visual availability of the object. Similarly, Eleanor and Dina began to recognize what qualities of one’s body motions must be highlighted and played with in the creation of graphical expressions because they are visible to the tool (e.g., moving one’s arm toward and away from the tower) and what aspects have to be part of a performance style (e.g., holding the button in a certain way, keeping yourself at a reasonable distance from the tower).

The process of sorting out what is idiosyncratic (what one needs to comply with for the use of the tool) and significant (what kind of actions one can enact to control graphical responses) embraced not only Eleanor’s experimentation with the tool but also her interaction with Tracey. For example, turning the button away from the tower might be used as a practical method to create vertical segments in a graph, hence it could be a significant aspect in the creation of graphs. The interaction with Tracey, however, induced Eleanor to perceive the button’s orientation as a peculiar feature necessary to comply with to achieve good performance from the tool. Developing and adopting the tool’s point of view seems to entail an anthropomorphization of the tool, such as the talk about the tower’s seeing the button or the sense that the tool is partially responsive to one’s actions, as other human beings are.

*The emergence of logical necessity.* As Eleanor enriched her conception of the tool as a point of view, she articulated a sense of logical necessity that allowed her to discern between possible and impossible graphs. For Eleanor, the inexorable graphical displacement to the right made vertical lines impossible unless one “kind of stop[s] the time.” For Dina, it implied that some letters are possible to graph and others impossible. Letters are familiar objects to Dina;

however, in her interest to draw them, she needed to look at them from a new perspective. Just after she drew the graph in Figure 10, she developed a taxonomy in which a capital letter  $T$  and a small  $a$  are impossible letters to draw with this device, whereas  $L$  or  $N$  are possible. By grappling with how to use the tool to draw letters, Dina encountered logical constraints that made apparent to her differences between drawing and graphing. Their emerging sense of logical necessity expressed simultaneously notions about the behavior of the graph (e.g., that it always goes to the right) and about the nature of body motion (e.g., that one cannot get farther away from the tower in no time).

It is this perception of how the tool “should” work that first suggested to us the notion of “adopting a tool perspective.” The three ideas that we have discussed—responsiveness of the tool, the tool as a point of view, and the emergence of logical necessity—were all important aspects of Eleanor’s and Dina’s development of a tool perspective. Our intent is not to talk about tool perspectives as mental objects but as interpretive frameworks that one uses to reflect on graphs and actions. Tool perspectives emerge from one’s experiences with the tool—experiences that include conversations with others, successes and failures in achieving intended results, and styles of performance. Developing a tool perspective that distinguishes between idiosyncratic and significant aspects of the motion detector’s behavior enables the tool user to imagine thought experiments that combine logical necessity and empirical evidence. Such a perspective can, for example, help someone imagine how other graphs of distance versus time would look, such as that corresponding to a car on a highway, notwithstanding that the range of distances may be much beyond what the tower senses.

## FUSION

Graphic displays thus not only provide physicists with a cognitive domain to inhabit and wander in, they also transport physical phenomena into the perceptual presence of physicists and serve as a locus in which physicist and physical phenomenon can be brought into symbolic contact with one another. Cognitive and gestural orientation to a graphical representation, therefore, make possible for physicists to symbolically participate in the physical events represented by the graphic space. (Ochs, Gonzales, & Jacoby, 1996)

A common way of talking about the understanding of symbols suggests that knowing the meaning of a symbolic expression is being able to recognize its correspondence to whatever it may refer to; for example, knowing the meaning of the word *chair* implies the ability to point to the objects that *chair* references. This image of correspondences between symbols and referents insinuates that understanding a graph of position versus time would be grounded in the ability to establish links between points on the graph and positions of the moving object at a given

time or between the slope of the graph at a certain time and the velocity of the object. Although the students we worked with learned to establish some of these correspondences (e.g., Eleanor's saying, "the closer to the tower it gets, the lower, I think"), we see this emergent ability as part of a broader process that we call *fusion*. Fusion is merging qualities of symbols with qualities of the signified events or situations, that is, talking, gesturing, and envisioning in ways that do not distinguish between symbols and referents.

We can observe some examples of fusion from Dina's first few minutes of work with the motion detector. During the following conversation, Dina was thinking of the height of the graph as showing speed rather than distance (see Figure 11).

Dina: On this one [Segment a] I kind of tried to think on how fast it would take to get slower. That's why I kind of let this one get slower slow [Segment c], and this one [Segment a] kind of fast. They look like birds, kind of.

Tracey: They do. And this wing [traces Segment c] gets slower slowly, and this one [traces Segment a] gets slower fast. And how about here [traces Segment b]?

The conversation between Dina and Tracey reflects a shared fusion experience. Dina's remark about the graph resembling a bird prompted Tracey to describe the wings. Invoking Dina's interpretation of graphical height representing speed, Tracey gestured the downness of Segments c and a as displaying their "get[ting] slower." Note the complexity of Tracey's pointing act: In a single utterance she

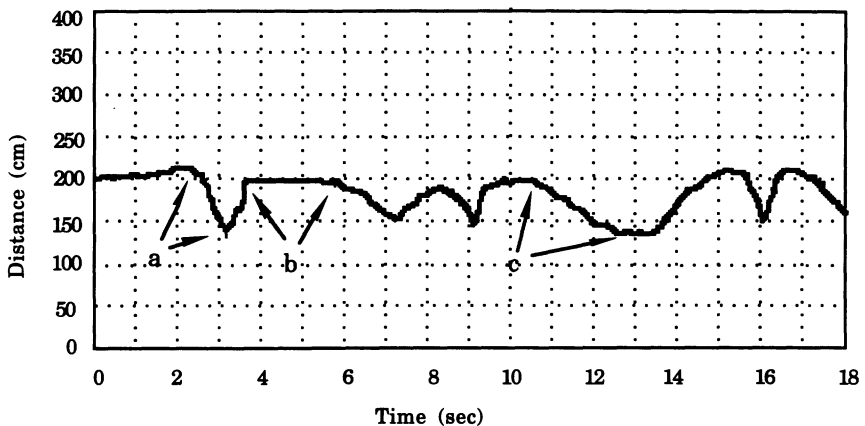


FIGURE 11 Dina: "On this one I kind of tried to think on how fast it would take to get slower."

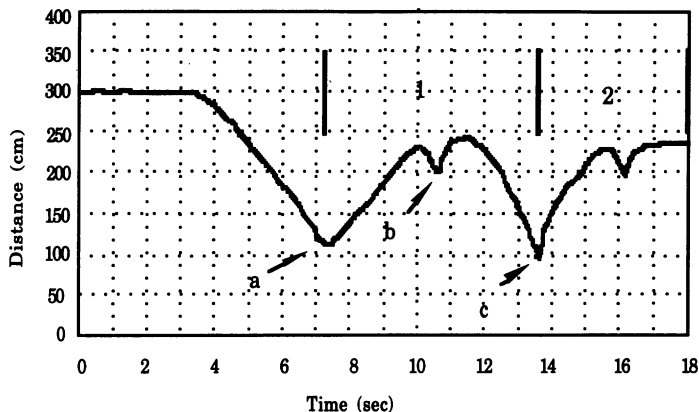


FIGURE 12 Dina: "I didn't really want to stay down at the bottom."

integrated the bird, the directionality of the graph, and a kinesthetic pattern. Tracey was pointing at all of them.

Dina begins her next graph (see Figure 12).

Tracey: I was noticing too that you've got a really sharp point down here [Point c].

Dina: I didn't really want to stay down at the bottom.

Who is "I" in Dina's answer? Dina knew that she, in the sense of her body in the room, had never been "down at the bottom," and yet her utterance makes sense to Tracey and to herself. The "I" that Dina invoked simultaneously moved the button back and forth and the graph up and down the computer screen and did not want to stay down at the bottom.

Tracey: Do you remember what you did with your hands here [after Point c]?

Dina: [referring to the upward piece] I just moved it up really fast, but close.

Tracey asked Dina about her hand motion. Dina's response describes a movement up and fast. What was the "it" that moved up and fast? Her description fuses a quality of the graph (upness) with qualities of her hand motion (closeness). Dina's statement is ambiguous, but her ambiguity is not a matter of vagueness or uncertainty; rather, it is an expression of connectedness and understanding.

Fusion involves the construction of a discourse in which indexical terms, such as *I*, *it*, or *here*, are constantly revised, dissolving boundaries among all the aspects that are relevant to the graphers' experience. The episodes with Eleanor and Dina

exemplify that this discourse incorporates not only body movements and graphical shapes but also intentions (“I didn’t really want to stay down at the bottom”), evoked objects (“They look like birds, kind of”), and past events (“Do you remember what you did with your hands here?”).

The idea of fusion is, we believe, more appropriate to the former examples than the metaphor of transparency. Learning graphing was not, for Eleanor and Dina, a matter of the graphs becoming transparent because, for them, the visual attributes of the graphs remained present and salient (e.g., “They look like birds”). Transparency implies that the graphs disappear from sight, enabling the grapher to grasp what is “behind” them. Fusion, instead, suggests that the qualities of the graph merge with the qualities of the represented events in ways that they cease to be distinct.<sup>7</sup>

In previous presentations in which we included the notion of fusion, we have noticed two reactions from members of the audience that we want to dispute here. The first reaction is that fusion expresses students’ inability to separate symbols and referents, that is, to put it more bluntly, students’ fusion indicates their conceptual confusion. We believe that this view is simply false. Eleanor or Dina could distinguish between the shape of the graph on the computer screen and their actual body motions across the room. Whenever the ambiguity of their words was an obstacle, they could spell it out. Fusion, in our view, does not constitute a misunderstanding but an achievement. By coming to see body motions, past actions, and evoked objects in their graphs, Eleanor and Dina crafted a rich background of intuitions and insights nurturing their interpretations. This conclusion does not imply, obviously, that whatever form of fusion they attained was error-free and correct. For example, Dina initially saw graphical height as an expression of the speed of her body motion. This surmise offered her an interpretive framework; it was in the context of seeing variations of speed in graphical shapes (“Because I think that low is slow”) that Dina began to sense the need to account for the distance to the tower in her graphical productions (after failing to reach the upper part of the computer screen, she says: “Well, you have to get far”).

The second reaction is that what we see as fusion is “just a shortcut” with little relevance. For instance, instead of “I didn’t really want to stay down at the bottom,” Dina could have said, “I didn’t really want to stay close to the tower because the graph would have stayed down at the bottom of the computer screen.”

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<sup>7</sup>Note that by the metaphor of transparency or fusion, we refer to the *common experience* with transparency or fusion. So, a glass window, by virtue of being transparent, enables us to see what we might see in the absence of the glass. On the other hand, some authors use the term *transparency* in more elaborate ways. Lave and Wenger (1991) defined transparency as a relational balance between visibility and invisibility, so that a graph might open up certain “fields” of visibility and invisibility, which could encompass simultaneous attributes of the graph and the represented situation. This particular use of the word *transparency* seems to be more akin to what we mean by fusion.

From this point of view, the former utterance was preferred because it is shorter and, presumably, easier to produce. However, this shortcut theory overlooks what we think is the most crucial quality of the fusing experiences: playfulness. Playfulness is at the root of symbol use (Piaget, 1962; Winnicott, 1982). Imagine that one sees a child who is playing and using a stick as a horse; the child then lets the stick fall on the floor, saying “my horse is tired.” Would one say that the utterance is merely a shortcut because the child could have stated, “I let this stick fall down because it stands for a horse that is tired”? The child is aware that the stick is not a real horse, and yet, acting, talking, and gesturing as if the stick were truly a horse is at the essence of play, of the child’s ability to be a part of the make-believe situation.

Fusion is not an exceptional or anomalous phenomenon. On the contrary, it is ordinary and pervasive. It may take place in using a map to explain to a friend how to get somewhere, in reading a poem in which the sound of the words is a crucial aspect to what they come to mean, in a religious ritual, or in a conversation about the characters of a cartoon. However, the experiences of fusion can be radically diverse. The type of fusion that emerges in the interpretation of a graph is not necessarily similar to the ones experienced in discussing directions on a map, reading a poem, or imitating a cartoon character.

The goal here is not just to show how fusion took place in the episodes with Eleanor and Dina but, more important, to gain insights on the specific forms of fusion that they experienced, which suggest important features inherent in the learning and use of graphing. Through the analysis of the ensuing episodes, we characterize three traits that are specific to the form of fusion developed by Eleanor and Dina and, we conjecture, to graphing:

1. The interplay between the graph as a shape and the graph as a response to actions,
2. The interplay between graphing and drawing, and
3. Imaginary traveling along trajectories on the graphical plane.

The study of physicists’ use of graphs by Ochs, Jacoby, and Gonzales (1994) and Ochs et al. (1996), and that we consider to be seminal, suggests that these characteristics of the form of fusion enacted by Eleanor’s and Dina’s graphing are ingrained in the practices of all novice or experienced graphers:

Through verbal and gestural (re)enactments of constructed physical processes, physicist and physical entity are conjoined in simultaneous, multiple constructed worlds: the here and now of the interaction, the visual representation, and the represented physical processes. These indeterminate grammatical constructions along with gestural journeys through visual displays constitute physicist and physical entity as coexperiencers of dynamic processes. (Ochs et al., 1994, pp. 17–18)

## Making Patterns

This section describes 4 min of Eleanor’s experimentation with the motion detector (see Figure 13). As soon as Eleanor had a sense of how the motion detector responded to her actions, she expressed her desire to use the tool for a purpose.

Eleanor: OK, I’m going to try to make a pattern. [She alternately walks forward with her hand toward the sensor then walks back away from the sensor. She doesn’t back up as far each time. As she moves back and forth, Eleanor watches the graph forming; see Figure 14.] Actually [right after producing seven peaks] this is not exactly the same pattern.

As her graph came into view, Eleanor noticed that it showed a shape different from the regular wavelike pattern she intended. This type of comparison between the actual graph and the pattern that she intended to create is a prevalent element that Eleanor, as well as all the other children we have worked with, brought to her graphing experience. These are moments in which the graph is seen as a shape that is assessed on its closeness to a “target” shape. We call this mode of graph interpretation *graph as a shape*. We shall now trace how the graph as a shape interrelates with another focus of graphical interpretation: the graph as a response to actions.

Tracey: Wow. Oh, I like that one. Wait, don’t clear it yet. Let’s stop it and look at it for a minute. What was happening?

Eleanor: Well, I was going far [tracing it with her finger as if redrawing the graph]. I was going like far, and a little bit closer but still far away then. I was really going like this [Eleanor moves her arm in a back-and-forth motion] but kind of changing a little [not as far away each time].

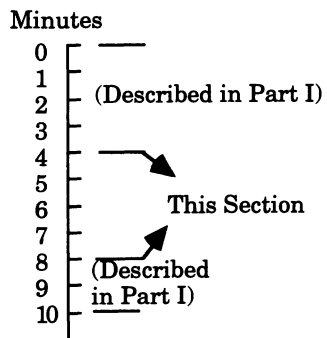


FIGURE 13 Time scale of minutes from the moment at which Eleanor started to use the motion detector.

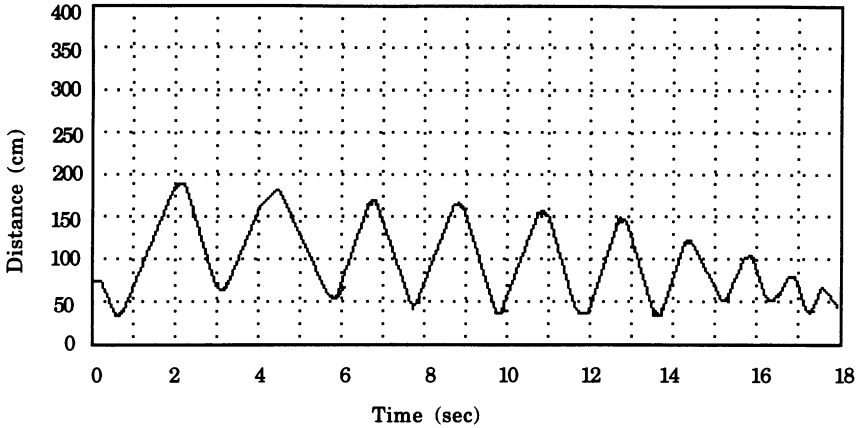


FIGURE 14 Eleanor: "OK. I'm going to try to make a pattern."

Tracey's question ("What was happening?") shifted the conversation to the graph as a response to actions; that is, it prompted Eleanor to reflect on the body movements with which she had created her graph and how they related to the graphical response on the computer screen.

Tracey: So the line up was when you were walking [how]? [Tracey points to one of the upward slanting lines.]

Eleanor: When I was walking backward [she gestures toward the back wall], and the line forward [she points to a line with downward slope] was that way [looking toward the tower].

This exchange elicited a kind of talk and gesturing that makes explicit the phenomenon of fusion of the shape with the actions. "The line forward" is the downward line on the computer screen as well as her walking forward. Tracey and Eleanor talk and gesture about an entity that is at once on the computer screen and in Eleanor's walking across the room, an entity that is present there as Eleanor points at it, but it is also past: "was that way." Eleanor could have said, adopting Tracey's language, "the line down was when I was walking forward," which is, seemingly, a "clearer" expression. However, we think that Eleanor's spontaneous use of "the line forward" expresses a common and significant aspect in graphing. The graph, the symbolized events, and the multiple graphers' interpretations are all experienced within a graphical space, in which they coalesce. The emerging space that Eleanor and Tracey experience, the one that they talk and gesture about, is common (borrowing from Latour's, 1986, common place) to both of them in several senses: It is shared by both of them, it is populated by events of which they are

mutually aware, and it blurs boundaries between the graph as a shape and the graph as a response to actions.

Tracey: The whole thing has sort of a shape too, doesn't it?

Eleanor: Yeah, it's all, like zigzags through the side [she turns her head sideways and does a zigzag motion with her finger], but I mean they're all. They look like, kind of like mountains or something.

Tracey: They do.

By alluding to the overall shape of "the whole thing," Tracey made salient this graph as a shape, which Eleanor then describes as "like mountains."

Eleanor: At first I was going to have it stay on this line [tracing her finger on a horizontal line from the top of the highest zigzags], but they got. They kept on getting smaller, so.

Tracey: Why did they get smaller?

Eleanor: Because I. I didn't walk as far.

Tracey's question turned the conversation again toward the graph as a response to actions. Tracey's "why" prompted Eleanor to analyze causes for the shrinking of the mountains and to consider how to maintain the height of the mountains throughout the graph.

Eleanor uses a note pad to mark a place on the floor where she would change direction.

Eleanor: Maybe I can mark where I walked to. And then like. Maybe I can like just put this like right here. [She places a note pad on the floor around 2 m from the tower.] OK. Can I make a new one [a new graph on the computer]? (...) Maybe if I like start here [she goes near the sensor]. No, if I start here [she walks back and stands on the pad] it will be like up [at the beginning of the graph]. [See Figure 8; Eleanor creates a graph by walking six times at a steady pace up to the sensor and back, stepping on the pad each time. The seventh time she steps back beyond the pad.] (...)

By marking a place on the floor, Eleanor did not need to look at the computer screen to reach a certain distance that her experience with the tool allowed her to estimate. She had developed an experiential sense of "being here" in which "here" meant simultaneously a place on the floor and on the computer screen.

Eleanor: [looking at the graph in Figure 8] That one [the last peak] went up a little too high, but ... [touching each of the peaks on the graph]. That

one [this graph] was kind of more the same [each peak is almost the same as each other] but a little bit different [from each other]. Because I didn't always, also I didn't always go in, in the same place [pointing to bottoms of each peak, which are slightly different], but it goes, tsh, tsh, tsh [touching the tops of the peaks].

Eleanor's comments on the graph in Figure 8 combined aspects of the graph as a shape and the graph as a response to actions. Throughout Eleanor's explorations, both perspectives were intertwined and under mutual influence. In this episode, Eleanor first wanted to make a regular pattern. She then envisioned a way of walking that would engender such a shape and created the graph in Figure 14. Noticing that the pattern was not regular, she thought of ways to adjust her body actions to achieve the missing regularity in the shape, an issue that she assessed after producing the graph in Figure 8.

## Discussion

In examining the former episodes, we have paid attention to how aspects of Eleanor's graphing characterized specific forms of fusion that she experienced. This discussion centers on three traits that are woven throughout the explorations of Eleanor and Dina.

*The interplay between the graph as a shape and the graph as a response to actions.* A conspicuous trait of Eleanor's and Dina's fusion experience throughout their conversation with Tracey was the merging of qualities of the graph as a shape and the graph as a response to actions. One example is Eleanor's utterance "and the line forward was that way," by which she blended a quality of her body motion (forward) while she gesturally stressed a quality of the graphical shape (downward). What Eleanor was pointing at was not on the computer screen or in the room but was both at once; it was not in her present or in her past but was simultaneously both.

These two perspectives for the planning and interpretation of graphs—graph as a shape and as a response to actions—are not independent; on the contrary, they influence and permeate each other. Sometimes the perspectives are fused in language use. At other times, shifts back and forth between the two perspectives are embedded in the language of intentions and causes, as in the vignette with the zigzag when Eleanor expressed how the resulting graph as a shape differed from the regular pattern that she intended ("They kept on getting smaller"), and Tracey's question ("Why?") led Eleanor to explain it by resorting to the graph as a response to actions ("I didn't walk as far").

Through their talk and actions, Tracey and Eleanor or Dina were developing a graphical space in which the absent is made present and in which their language integrates shapes on the graphical plane, body motions performed in the room, and intentions held by the graphers.

*The interplay between graphing and drawing.* A resource that Eleanor and Dina brought to their graphing activities was their drawing experience. Interpreting the graph as a shape is, in a way, looking at the graph as if it were a drawing.

Eleanor's and Dina's drawing experience was a resource for them because it was a source of ideas (Dina: "Do you know anything with two lines? Electricity poles!"), ways of talking (Eleanor: "That one [the last peak] went up a little too high"), and styles of appreciation (Dina: "You could make a nice picture with all the colors"). They expressed the relevance of drawing by invoking imaginary objects whose silhouettes resembled graphical shapes. This aspect was particularly important to Dina: During the creation of a graph in which Dina and Tracey had intended to draw electricity poles and wires, Dina commented: "When we were still about here, I was thinking we could have made an elephant if we went straight down." In Dina's conversation with Tracey, the elephant became a useful means for the mutual recognition of the different parts of the graph (Dina: "But then it would have no front legs. And if it had front legs it would have no back"), for planning their body actions (Tracey: "So if you did the legs and I did the back, then what would I do"), and for assessing results (Dina: "Oh geez, that's not a leg").

Another domain in which they expressed this enrichment was gesturing. Drawing nurtures a type of symbiosis between shapes and gestures. The act of drawing a certain shape is a way of gesturing it and is part of what the gesture comes to mean in a situation (e.g., a smooth wavelike shape and a gradual hand motion that traces it may evoke each other and elicit common connotations, such as a calm and slow rhythm or a steep line and the abrupt upward or downward hand motion that gestures its sudden change). The drawing experience was also important because it enabled Eleanor and Dina to notice ways in which graphing was different from drawing, such as when Eleanor concluded that making a vertical line was a problem ("You would have to stop time") or when she realized that a mark on the floor would help her to level off all the peaks. As Dina and Eleanor realized that something might be easy to draw but impossible or difficult to graph, they broadened and refined their drawing-based expectations.

*Imaginary traveling along trajectories on the graphical plane.* Often ideas articulated by Eleanor and Dina included descriptions of trajectories on the graphical plane; these used verbs, such as *to go* and *to stay*, that suggested forms of traveling across it.

Eleanor: I wonder if you could get it to go straight up?

Eleanor: At first I was going to have it stay on this line [tracing her finger on a horizontal line from left to right].

Dina: We could go a little higher and then go straight down and then go up.

The graphical plane serves as a pointable record on which trips can be planned or reviewed. How the graph is generated over time defines the temporal sequence of the trip: The trajectories always unfold from left to right, and they never “go back.” Even though the completed graph showed a trajectory that was all present at once, Eleanor and Dina learned to project on it a sense of time rooted in their experience of the genesis of the graph across time. The envisioning of trips across the graphical space seems to be a crucial aspect of the experience of fusion; it enables the graphers to create narratives (Nemirovsky, 1996) that express simultaneously the shape of the trajectory and the events that the graphers imagine would take place along the way. One further example took place during Dina’s use of an electric train with the motion detector button taped onto it (see Figure 15). She attempted to make a horizontal line, and it did not work.

Dina: I kind of messed up. I had to leave the train straight.

Tracey: You had to what?

Dina: Leave this train without moving it.

Tracey: To make?

Dina: That [pointing to a horizontal line].

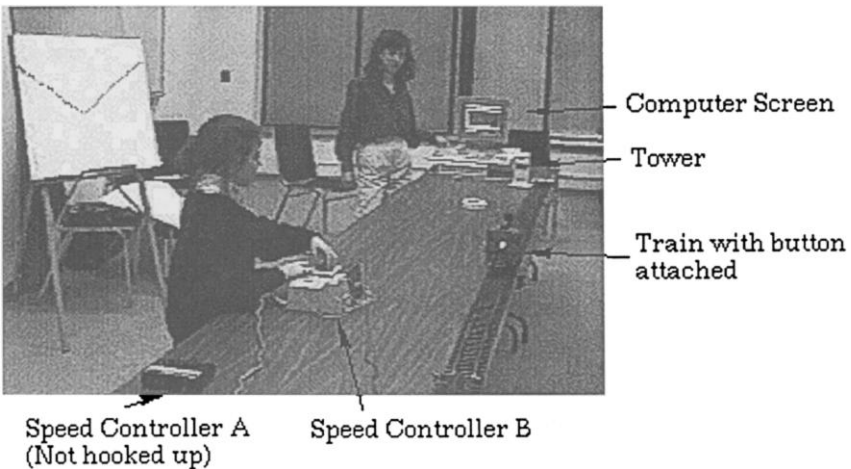


FIGURE 15 Dina and Tracey using the motion detector and electric train.

Dina explained how she had “messed up” to Tracey by using a remarkable expression of fusion: “I had to leave the train straight.” Moreover, demonstrating the difference between fusion and confusion, she clarified her point: “Leave this train without moving it.” Eleanor and Dina and others we worked with were quite consistently able to separate the qualities of the graph as a shape from the graph as a response to actions if the situation required it.

We conclude this discussion by describing a short “last minute” activity that took place at the end of the session with Dina because it introduces the theme of graphical spaces. Tracey asked Dina to create a graph using two buttons simultaneously and consequently to generate two line graphs at the same time. Dina moved her arms back and forth to generate crossing waves of small, then larger, and then smaller amplitude.

Dina: I thought of getting bigger, bigger and then smaller and smaller (...).  
I kind of ... just thought of skiing [Dina gestures her arms swinging forward and back] or something.

Holding one button in each hand prompted Dina to experiment with a new kinesthetic behavior: swinging her arms forward and back, keeping them in opposition (one arm is behind her when the other one is in front, and vice versa). In this mode of body motion, what was salient to her—the feature to play with—was the amplitude of the wavelike graph. Dina’s work with the motion detector provided many examples of figurative interpretations of the graph (elephants, birds, etc.), but in the preceding excerpt, we also find an example of an interpretation based on a *kinesthetic pattern*: “I kind of ... just thought of skiing.” Skiing was not an image shown by the graph but by the regular body movement with which Dina had created the graph. Kinesthetic patterns were significant elements of the graphical space that became particularly noticeable for us when the use of the tool was changed in conspicuous ways, such as from holding one to holding two buttons at once.

## GRAPHICAL SPACES

The graph on the screen is not merely something to be looked at, but instead an open gateway to a world where the human body can move and act within new frameworks of meaning. Like a playing field that builds a landscape within which certain moves, such as a “goal,” become both possible and visible, the graph on the computer screen creates an arena for the perception and constitution of relevant action. (Goodwin, 1995, p. 257)

Eleanor and Dina learned to create graphs by getting closer to or farther from the tower with the handheld button. The episodes that we analyze in this section were at first puzzling to us. A change in the way of using the motion detector

prompted Eleanor and Dina to explore the tool anew, apparently shifting them back to the methods with which they had first investigated the tool. The change for Eleanor was her use of two buttons instead of one, holding one in each hand, so that she could generate two graphs at once, a yellow one and a blue one, corresponding to the respective colors of the buttons. For Dina, the new feature was to attach her button to an electric train, which she manipulated using a speed control device; this graph showed the motion of the train (rather than her body motion). Both girls treated the motion detector in the new contexts as if it were a different tool from the one they had used. Eleanor started to investigate “what matters,” that is, what kind of motion elicits graphical responsiveness, whereas Dina went back to her original idea that the graph showed the speed of motion (as in “low is slow”). We shall see that, as they progressed in their analyses, Eleanor and Dina went through moments of remembering, which brought back their experience with the motion detector in the original context of one handheld button.

Why did Eleanor and Dina go through such a process? Why did they not directly see that using two buttons and attaching the button to the train are variations that preserve what the graph is responding to, namely, the distance of the button from the tower? Some readers may be tempted to explain Eleanor’s and Dina’s temporary “regression” by invoking matters of ignorance, such as “they had not really understood the motion detector and the graph of distance versus time” or “they had grasped only the superficial or situation-specific behavior of the computer graph.” However, these kinds of explanations are futile: What do we gain by saying that someone did not do X because she did not know X? What counts—what may help us learn something new—is to develop a deeper understanding of the qualities of their new experience that made Eleanor’s and Dina’s actions sensible. Our research has something to contribute to the extent that it nurtures in the reader a richer view of how the situation looked from Eleanor’s or Dina’s perspective.

Our analysis makes extensive use of the centuries’ old distinction between subjective space and time and objective space and time. Everyone understands that the same minute, as measured by a clock, can be felt as an eternity in some situations and as a fleeting instant in others. Subjective space and time are sometimes called *intuitive*, *experiential*, or *lived*, whereas objective space and time may be called *metric*, *Newtonian*, or *absolute*. We refer to them by using the words *lived-in* and *metric*, respectively. Metric space is the notion of an infinite, regular, quantifiable space. The lived-in space, instead, is forged on an ongoing basis: through our actions, expectations, and life history.

The traits of lived-in spaces become noticeable in situations of change, such as when changes are made in a very familiar place; for instance, a prominent table is removed from a room that we frequently use. When we enter the room, we immediately notice that it is not the same room we are accustomed to. Sometimes, we do not easily recognize what has specifically been changed; even when we

identify the missing table, the room looks different, possibly bigger and emptier. As time goes by, as we start to live in this altered room, it becomes just what it is, not bigger or smaller, emptier or fuller, but a “normal” room. This gradual transformation of the room becoming ordinary is not a change in the metric properties of the room but in our experience of the room, how we walk in it, what activities we develop there, and our daily reencounter with what has remained in the room.

From this example, we want to highlight three properties of lived-in spaces, namely, that they are relational, intentional, and creative. By relational, we mean that changes, even if they are physically circumscribed to a particular aspect, affect the lived-in space as a whole.<sup>8</sup> Lived-in spaces are intentional in the sense that they are places to do things and to accomplish purposes. A room without a table may not be a good place to have dinner anymore, although it might be an appropriate place for doing a somersault; the kinds of activities that one performs in the room are an essential aspect of what it becomes as a lived-in space. Finally, by saying that lived-in spaces are creative, we emphasize that they are not set and fixed but always subject to and constituted by the ongoing drift of the life experience.

We argue through our analysis of the subsequent episodes with Eleanor that, for both girls, the graphical space was a lived-in space: Eleanor experienced the addition of a second button; Dina, the attachment of the button to the train, in ways roughly analogous to the former example with the missing table. These changes involved an adjustment in the type of actions that they performed with the motion detector as well as in the expected responsiveness of the tool. They did not see immediately what had changed and what had not changed. As they explored the new setups, they reencountered previous experiences with the single handheld button, events that they both described with the words “I remember.”

Central questions opened by this analysis are about the relation between lived-in and metric spaces. Some may argue that the graph of distance versus time represents the distance between the tower and the button as a metric space and that only when this is conceptualized does one truly understand the graph. Therefore, do lived-in and metric spaces constitute a fundamental dichotomy? Are the metric spaces the ones for mathematics and the lived-in spaces the ones for poetry? At this point, the reader may have little difficulty guessing that our answer is no, and that far from a dichotomy, metric spaces become meaningful when one participates in the more general experience of lived-in spaces. To clarify this idea, the following example may be helpful:

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<sup>8</sup>This phenomenon of seeing changes holistically happens all the time. One of us was recently surprised to meet a colleague who looked different because he had changed “something” in his face. Asked to guess what had happened, the hesitant response was that he had grown a mustache. It turned out that he had shaved his beard.

Every person who comes to the United States from a country in which the temperature is normally given in Celsius degrees encounters the problem of making sense of the Fahrenheit scale. Being taught the definition of the Fahrenheit scale is not necessarily helpful, even if one is well aware of its history and its two critical points. The information that the temperature will be, say, “between 65 and 70 degrees Fahrenheit” is meaningless in how the weather will feel, what to wear, and so on. At first this person “translates” degrees in Fahrenheit into the Celsius scale. By translating these numbers into the Celsius scale, a whole background of expectations comes to the fore. As time passes, however, and this person continues to live in an environment in which Fahrenheit degrees are customarily used, the Fahrenheit scale starts to have a meaning of its own, and gradually the need to translate vanishes. We say that the person’s experience with the weather is gradually *indexed* by Fahrenheit degrees.

Through the former example, we want to suggest that metric spaces are not free-floating ideas held together exclusively by formal definitions. Indexing our experience with a metric space is a thoroughly situated process. H. Sacks (1995) developed an analysis of this situatedness for the case of speed. He analyzed the expression “John likes to drive fast” and pointed out that this expression is not made more accurate by specifying the number range for the speed at which John likes to drive. What is being said is that John drives fast in relation to the traffic situation, the shape of the road, the kind of car he drives, and so forth; in some circumstances of heavy traffic, driving at 30 miles an hour is dangerously fast, and in other circumstances, as on an empty highway, it may be banned for being too slow.<sup>9</sup>

This situated relation between metric and lived-in spaces has been eloquently described by Berg (1970). To explain why “a meter *here* is never one meter *there* [italics added]” (p. 16), he related a hiking experience from his youth. He and his companion were exhausted after several days of hiking when they saw a sign indicating that the distance to their final destination was “5 Km.” The sign was overwhelming to them. They felt that the road they still had to traverse was extremely long; 5 km appeared to them as an almost insurmountable distance. After pointing out different factors that made the road so long (hunger, tiredness, thirst, etc.), he said:

Perhaps other factors would have made it shorter. Factors are never absent. For the length of the road to remain constant, a very large number of factors would have to be fulfilled. The hiker should then be neither well-rested nor exhausted, neither happy

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<sup>9</sup>On August 22, 1995, the *Boston Globe* reported on speed laws: “Local drivers pose a variation of an oft-asked hypothetical question: If a car speeds across a state and no one is around to see it—not any police officers, not even any other drivers—is there really such a thing as traveling too fast?” (p. 8)

nor sad, neither hungry nor satiated, neither lonely nor with others. But there is no such a person. ... Only for those who are absent the dimension of things remain constant. In practice this means that one who wishes to perceive with constant dimensions must absent himself. Since this is not possible—which implies that constant dimensions are impossible—the rule is that one who wishes to perceive with maximally constant dimensions must absent himself as much as possible. (pp. 18, 20)

Note that the view that metric spaces become meaningful by indexing lived-in spaces is different from the traditional notion that metric spaces are abstractions, in the sense of expressing the isolation of a specific aspect or property from a complex object.<sup>10</sup> Saying that the statement “today’s high temperature will be 65 degrees Fahrenheit” abstracts a particular property of the weather does not help one understand why the newcomer to the United States finds it useless to know the type of weather, even though he or she knows that it defines the temperature.<sup>11</sup>

Our analysis of the relevant episodes with Eleanor and Dina highlights how they began to develop a sense of metric space by indexing their experience with the motion detector. This process started at the very beginning of their use of the motion detector. Eleanor quickly noticed relations, such as “the closer I go, the lower it goes,” and developed a sense that her closeness to the tower was indexed by the height of the line on the graph. Later, she used this qualitative relation to mark an exact position on the floor and help herself remain within a certain range on the graph. Dina, on the other hand, started expecting that what was being indexed by the graphical height was her speed (“low is slow”). In this context, she used the numbers on the vertical scale.

Dina: And here [pointing to the 50 mark on the vertical axis] it also shows, it says 50 down below.

Tracey: So you think if you moved it really fast, then?

Dina: It would be at about 350 [on the vertical scale].

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<sup>10</sup>Arnheim (1971) distinguished two ways of conceiving what an abstraction is. The first one is the traditional notion of abstraction as a detachment and isolation from the complexity of a situation. The second one, which he endorses, is the recognition of patterns fully embedded in the idiosyncrasies of a lived situation. Our view on indexing is consistent with the latter.

<sup>11</sup>On October 1, 1995, *The New York Times* reported on new steps taken in England to impose the use of the metric system. The following excerpt illustrates the difference between a metric space as an abstraction or as an index:

But shopkeepers themselves are distraught. “I can feel this cloth, I know it’s 13 ounces,” said John Davis, a partner in Tobias Tailors on Savile Row, rubbing a Prince of Wales check from a patterns book between his fingers. “How can I do this in grams? I can’t. ...” Greta Hutsheson, owner of Lucy’s delicatessen in Mayfair, said, “I feel resentment at the change.” She found it hard to visualize servings of pasta, beans, salads, and fish. “I can do the conversion in my mind but I don’t get a mental picture of how big it is.” (p. 6)

Here, we trace how the shifts in Eleanor’s way of using the motion detector led her to grapple with new lived-in spaces, that is, new kinds of actions and graphical responses, and how, as part of this development, she broadened her perception of what was being indexed by the height on a distance versus time graph.

The episode presented in this section spans from Minutes 14 to 21 (see Figure 16). Between Minutes 8 and 10, Eleanor had explained that vertical lines were impossible because time goes on “no matter what.” She then produced a graph to show that one could get closer to the vertical if one went faster. Between Minutes 10 and 14, Eleanor made high near-vertical graphs, and then she explored the meaning of “pointiness” on the graph.

Eleanor found that a way to create pointy graphs was by staying in one place and moving her hand holding the button back and forth very quickly. Eleanor said that the point was where one changed directions, and she created a few pointy graphs. She then played with the combination of walking and arm swinging to create the graph in Figure 17.

Eleanor: [at the beginning of Segment B] It went a little bit higher. Yeah, [she looks at the completed graph] so it can kind of get to go in a straight line. [Tracey: Mmm-hmm.] Like, well, here [Segment A] I was trying to get it to go between these squares like there [keeping the zigzags between the two height lines].

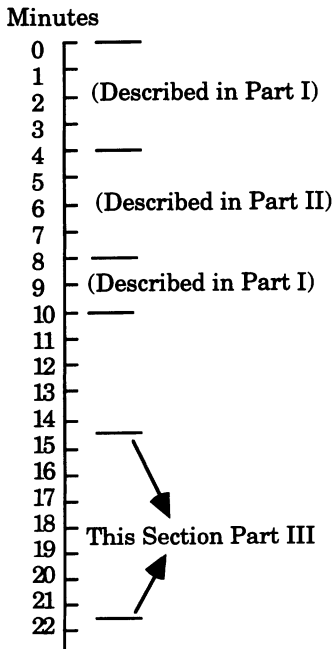


FIGURE 16 Time scale of minutes at which Eleanor started to use the motion detector.

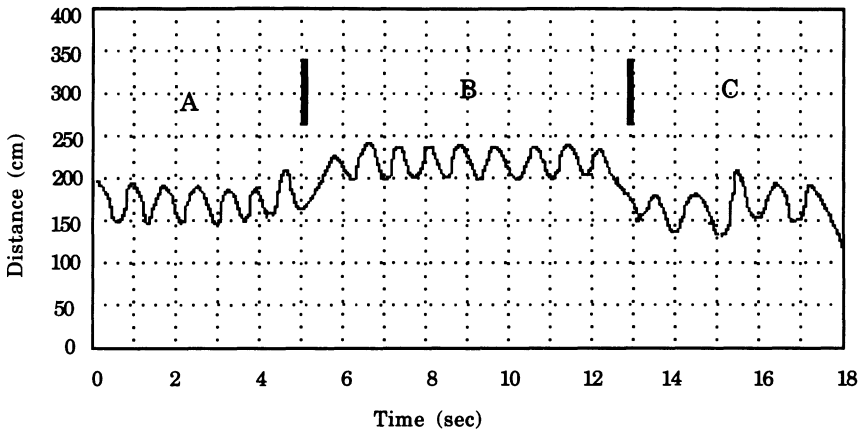


FIGURE 17 Eleanor: "I was trying to get it to go between these squares."

Tracey: What happened here [points at the transition between Segments B and C]?

Eleanor: Well, maybe, I don't know. I just kind of went—I went like this [she walks toward the tower] a little closer to it [the tower].

In creating this graph, Eleanor was playing with kinesthetic patterns that combined arm waving with going toward and away from the tower. Her graph displayed an average height that showed how far she was from the tower and a finer up and down that expressed her continuous arm motion. Then, Tracey introduced the second button. This is the point in time that we have chosen to begin the episode.

### One or Two Buttons

Tracey showed a second button to Eleanor and suggested that they could use both, one held by Eleanor and another by Tracey to create a new type of graph. First, Eleanor and Tracey each held one button and did a sort of improvisational dance together with Eleanor taking the lead and Tracey responding by moving her button sometimes alongside Eleanor's and sometimes in the opposite direction. Eleanor moved the button back and forth as well as up and down, making a diagonal trajectory. Eleanor commented that the color of her graph was yellow and Tracey's was blue, as indicated by the color of the buttons that each of them was holding. (In the following graphs, we show the yellow graph with a thick line and the blue graph with a thin one.) Then, Eleanor asked to have both buttons.

Eleanor: Wait, I want to try something with both buttons to see if I can kind of make them go right next to each other. (...) Maybe if you [I] hold

both of them. [Eleanor takes Tracey's button from her so that she is holding both, one in each hand. ] Like you put—I wonder, which one would be above [in the graph]? OK. I'm just going to do something just like that. [She walks back and forth holding the two buttons close beside each other in front of her, equidistant from the sensor; see Figure 18, beginning of Segment A. Then she holds them far apart one above the other then out to the sides, still equidistant from the sensor as she moves back and forth, Segment A. Then she stands still near the tower and moves the buttons alternately in and out, back and forth, toward and away from the tower, making an alternate zigzag, Segment B.] Wow, that looks cool. [Eleanor does larger swings in and out, Segment C. Still standing near the tower, she circles the buttons around one another as if pushing peddles on a bicycle, Segment D. Finally, she swings the buttons out to the side and in until they touch, Segment E. The graph stops.]

As soon as Eleanor held the two buttons, she began to experiment with relational motion framed by her new question: "Which one [line] would be above?" The actions that she had enacted with one button were different ways of making the line rise and fall (walking, running toward and away from the tower). Her new movements and questions, instead, focused on different ways to separate the lines. She first held the buttons together, side by side; then, she experimented with different ways of pulling the buttons apart, first vertically, then to the sides (Figure 18, Segment A), in attempts to separate the two lines on the computer screen. However, she did not achieve a conspicuous response on the computer screen; the

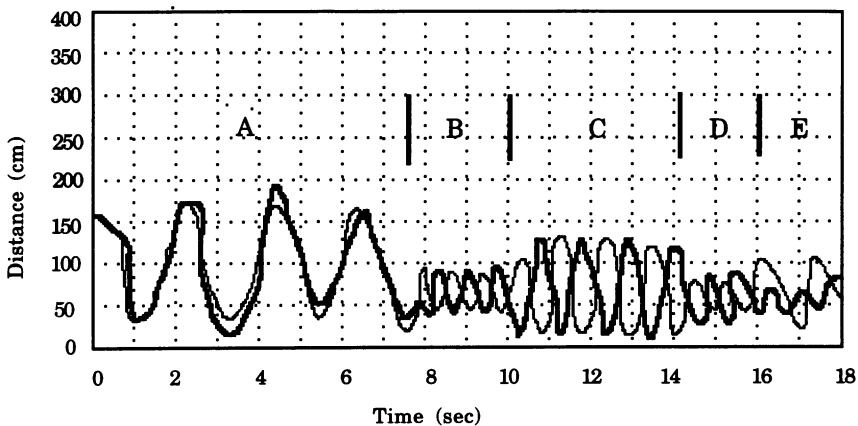


FIGURE 18 Eleanor: "I want to try something with both buttons to see if I can kind of make them go right next to each other."

lines were still together. Then, she tried separating the buttons by alternately moving one closer to the tower and the other back (Segment B) and immediately discovered a significant reaction when the two graphical lines moved apart. She emphasized the responsiveness of the computer by enlarging the swings of her arms (Segment C) and playing with different ways of circling her hands around each other, toward and away from the tower (Segment D). The two line graphs on the computer screen expressed to Eleanor different kinesthetic patterns of two-arm motion; this is the subject of the ensuing dialogue.

- Eleanor: That's neat. I like this one if you go like this [she points to the screen, Segment B]. Wait, I forgot what I was doing. I think I was going like this [she moves the buttons alternately back and forth]. That looks neat.
- Tracey: Yeah, that was that part [points on the screen to Segment B].
- Eleanor: Yeah, and this part's kind of weird [Segment E]. I think I was going like this [stretching her hands out to the sides then in together]. [Tracey: Right.] Wait, I was trying to figure out, which [way of pulling the buttons apart] makes them [she points on the screen to a vertical gap between the lines] go like farther apart? Like this [holding buttons apart to the side] or like that [holding one up and one down] or ...
- Tracey: Oh, OK, right. And at first [Segment A] you were holding it like that [holding one hand up and one hand down] and you were also wondering which one would be on top [pointing to Segment A]? Which line would be on top?
- Eleanor: Yeah, except here [pointing to the screen where lines are close together in Segment A] I was holding them like this [holding them together].
- Tracey: Right. And what happened [pointing to first part of Segment A]?
- Eleanor: They were just—you can't tell the difference [between the two graph lines].
- Tracey: And then here [second peak in Segment A], I was noticing you held this one [touching the yellow button] up high. But look [at second peak in Segment A], the blue one is up higher than the yellow one.
- Eleanor: Well, let's. I'm going to try it to see if it like matters if like height or like, if like it the yellow one's higher [holding yellow slightly above blue vertically] and just like have it [the graph] go straight across [horizontally]. [Tracey: OK.]

Notice how Eleanor started to explore the motion detector anew. The distance to the tower became inconsequential in her use of the tool. A new range of actions and goals came to occupy Eleanor's concerns and curiosity, all of them centered

on the relative distance between buttons and between the two line graphs. The kinesthetic patterns expressed in Segment A “failed” because “you can’t tell the difference [between the two graph lines].” Although Eleanor had found one style of arm motion that separated the two lines (in Segment B), she still felt that she did not know “what matters.” Eleanor’s movement back and forth in Segment B was for her a particular case of arm motion that “worked,” but she wondered if vertical separation of the buttons could also elicit the same response: “I’m going to try it to see if it like matters if like height.” Which button is higher in the air and which corresponding color on the screen is higher became her next question.

Eleanor: Hmm. [Tracey starts the computer. Eleanor holds the buttons vertically, in opposite positions again (one up, one down) to keep them still for a few seconds each time. Then, she stretches very far, one down to the floor and one up as high as she can reach. At 5 sec, she says] I wonder if I hold it like that [holding each one out to the side]. Hmm. I don’t know. [She goes back to holding them apart vertically (one up, one down), Segment A; see Figure 19.] They [the two line graphs] kind of go the same. [Tracey: Yeah.] [Left hand holding blue button forward and right hand holding yellow button back, Segment B] There, kind of found kind of a combination. [She sounds pleased, reverses her arms, Segment C, then alternates them quickly, Segment D. Graph stops.] Hmm ... that’s neat, so I was kind of holding them like this and like that. [Tracey: Right.] [Repeating previous motion from Segments B and C, she holds her left hand in front of the right then reverses arms so that her right hand is in front, all on the same height.] Like one’s [button] back and higher [gesturing with left hand] and one’s forward and lower [gesturing with right hand].

When Eleanor finally achieved a conspicuous separation between the graphs, she remarked “There, kind of found kind of a combination.” Note how she described her arm motion by highlighting that one was back and higher. Eleanor found again a successful kinesthetic action, but her recollection also included a component of vertical displacement. Does the height of the button matter or not? Her first action of putting the buttons up and down during Segment A suggested that the button’s height did not matter, and yet the “successful” Segments B and C appeared to be partially related to vertical displacement of the buttons.

Tracey: Right. And which part was that on the screen?

Eleanor: [She interprets the question as what she had been doing during Segment C.] I think when the yellow one [button] was back, it [the yellow graph] was higher, I think. [Tracey: Yeah?] Yeah, I remember the farther back [gesturing back from the tower] you hold it [pulling

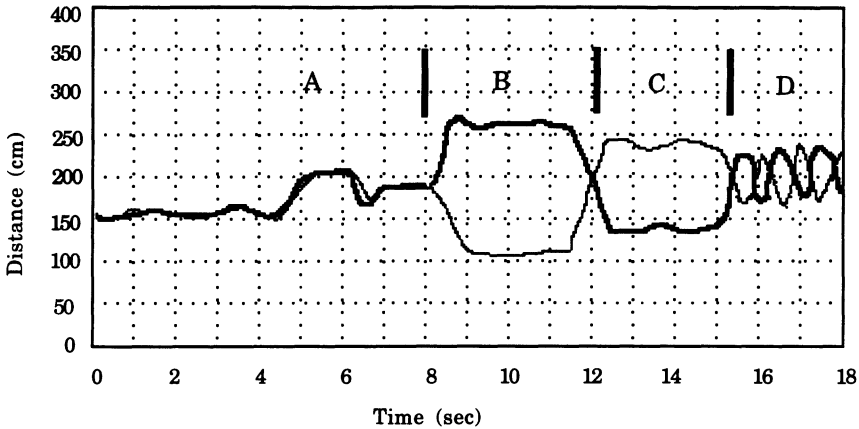


FIGURE 19 Eleanor: "I wonder if I hold it like that."

her right arm and body dramatically backward] the higher it is [gesturing upward with her right hand].

Tracey: Right, from the beginning [of working with the motion detector].

Right after Eleanor said, "I think when the yellow one was back, it was higher," she recalled memories of her former experience with one button, in which being back away from the tower meant being higher on the graph. This was a moment of reencountering previous ways of acting and interpreting the graph. She experienced a sudden remembering that cast light on her question of what separated the two graphs. Eleanor immediately stopped looking for different ways to achieve such a separation because it ceased to be a question for her.

Eleanor: I'm going to try to make a pattern like that kind of [Segment D].

The use of two buttons had prompted Eleanor to start an investigation very similar to her initial one with the motion detector. Now, feeling a new sense of control with two buttons, Eleanor reacted by challenging herself to generate visual patterns. The parallel with her actions in the theme of fusion is striking. Once she had established (in the theme of tool perspectives) the conclusion that "the closer to the tower it gets, the lower"—which enabled her to control the vertical displacement of the graph by walking back and forth—she said: "OK, I'm going to try to make a pattern."

Tracey: So in the beginning part [pointing to Segment A] you were experimenting with it (...) [gesturing at the same time as Eleanor].

Eleanor: I was holding it [the buttons] like this [moving one button up and the other down] and like that [out to the sides], but that didn't really work. But then I remembered that like when you held ... when you went back farther [pulling her arm back as she steps back] it went up higher [pointing up with her front hand as if drawing in the air]. OK, I'm going to try to do, like, patterns with them.

It seemed that Eleanor had temporarily “forgotten” that the distance to the tower was what counted. Why? Did she deem that by using two buttons the tool might work in entirely new ways? We think not; this type of interpretation would amount to a deliberate attitude that is inconsistent with Eleanor's ongoing surprise in meeting unexpected results as well as with her sudden remembering that seemed to settle the question as to what counted. In our view, what Eleanor experienced with the use of the two buttons was a new realm of possible actions and responses. The notion of a lived-in space implied that the kind of actions one performs and perceives constitutes the space itself; working with two buttons shifted Eleanor into a new space, so that her movements, gestures, and graphical productions came to be permeated with new kinesthetic and visual qualities.

For Eleanor, using two buttons was dramatically different from using a single one. Her whole body moved forward and back in consonance with the one button. With two buttons, she often stood still and moved her arms while holding the buttons; she thus became a separate observer of the relation between the buttons. As she had done at the time of her initial encounter with the motion detector, Eleanor proceeded to enact the universe of possible actions to test its limits and to assess its responsiveness. This does not mean that her previous experiences (in the themes of tools perspectives and fusion) disappeared, only that they did not seem pertinent to her new actions. They then become available to her through her new experience: “But then I remembered that like when you held ...”

This episode shows that a lived-in space is an ongoing creation; that far from being an inert structure, it demands a constant re-creation as one practices new possibilities and ways of acting in it.<sup>12</sup>

## Discussion

We want to articulate two points: that the graphical spaces were, for Eleanor and Dina, lived-in spaces and that the distance and time indicated by the graphical location were meaningful to Eleanor and Dina as indexes of their experience with the tool.

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<sup>12</sup>A discussion of related ideas typically described as “transfer” can be found in Schliemann, Carraher, and Nemirovsky's (1995) *Situated Generalization*.

*Graphical spaces as lived-in spaces.* Eleanor started out playing with two kinesthetic patterns, arm waving and walking, each one with a certain identifiable visual response (waves and overall height). As she used two buttons, she explored new kinesthetic patterns focused on the relative distance between her hands aiming at a particular graphical response: separation between the line graphs. She perceived a holistic change in her use of the tool. She wondered anew about what mattered. In a particular configuration of circumstances (the button more to the back had generated a higher graph), she remembered that “the farther back the higher.” Her insightful reencounter with her previous idea helped her gain a sense of control over the tool that she expressed with exactly the same words she had used previously: “OK, I’m going to try to make a pattern.” As she had done after figuring out how the tool worked (in the theme of tool perspectives), she moved to using the tool for the production of visual patterns.

In the case of Dina, she envisioned two distinct graphical spaces for the same graphical shapes: one about near and far, the other about fast and slow. When Dina held the motion detector button in her hand, she primarily interpreted graphical height as indicative of distance to the tower (although at first she conceptualized graphical height as speed saying “low is slow”). When the button was attached to the electric train that she moved using a speed control, Dina moved away from graphical height as distance and saw it as primarily representative of the speed of the train. It was the problem of how to make a flat line with the train that disrupted Dina’s simultaneously held perspectives. During her first few attempts at a horizontal line, she perceived this task as impossible to create with the train because it had to start at zero speed and get faster, therefore increasing height on the graph. As she was staring at the computer screen, however, she suddenly remembered that flat lines meant stillness, drawing on her previous experiences in holding the button.

Dina: [telling her plan to graph a horizontal line with the electric train] (...) Oh! I also remembered another thing on the computer. If I wanted to make a straight line on that [computer screen], I wouldn’t be able to move the train. (...)

Tracey: So if you want to put, to make a flat line with the train ...

Dina: I wouldn’t be able to move it [the train].

A minute later, she responded to Tracey’s question of where to put the train to start a graph of a flat line: “It depends ... how high I wanted it [the line on the graph].” For Eleanor and Dina, the space between themselves and the tower was the arena in which they created different lived-in spaces populated by specific intentions (e.g., keep the mountains the same height, separate the graphs, create a flat line) and different types of actions (e.g., hand separation, getting closer, leaving the train straight), eliciting diverse graphical responsiveness.

*Graphical locations indexing one's experience with the tool.* If we use "abstraction" in the sense of "the act of singling out" (James, 1983, p. 477) an element of reality in its relative isolation, we note that, for Eleanor and Dina, the distance from the tower was not primarily an abstraction. As they created new lived-in spaces, different forms of indexing emerged. At the beginning (of the theme of graphical spaces), Eleanor and Dina saw the height of the graph indexing their distance to the tower with the handheld button. The use of two buttons prompted Eleanor to open up the issue of what was being indexed by the separation between the two lines. As she described her body actions in creating the graph in Figure 19, Eleanor recognized that the relative separation between the buttons that mattered was the one that put them more or less close to the tower: "Yeah, I remember the farther back you hold it [the button] the higher it [the graph] is." Dina, on the other hand, saw the graphical height as indexing either her distance to the tower (when the button was in her hand) or the speed with which she drove the train. The case of the horizontal line, however, led her to preserve the distance to the tower as the primary indexing that matters. The stillness of the train was an important case for her. It is as if she were the train holding the button. Dina extended this form of indexing to the train when she told Tracey that where you put it depended on how high you wanted the graph.

The idea of distance as indexing one's spatial experience has been highlighted by several philosophers. Merleau-Ponty (1945/1989), for example, called distance the "dimension of inactuality" because nearness and farness emerge from the "hold" that one can have on the object.<sup>13</sup> Distance, he said, is about the impossibility of certain actions that would be possible if the objects were nearby, not a matter of meters. In other words, being near is primordially the possibility of touching, seeing "eye to eye," and all the other activities enabled by body proximity. Similarly, for Eleanor and Dina, distance to the tower was not primarily a matter of meters; it concerned the possibility and impossibility of creating certain graphical responses.

## CONCLUSIONS

Eleanor: You'd have to kind of stop the time.

Dina: I had to leave the train straight.

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<sup>13</sup>O. W. Sacks (1995) noticed that "it has been reported that if people who have lived their entire lives in dense rain forest, with a far point no more than a few feet away, are brought into a wide, empty landscape, they may reach out and try to touch the mountaintops with their hands; they have no concept of how far the mountains are" (p. 119). An instance of such a report is provided by Turnbull (1961); it includes the example of an Ituri forest pygmy looking at buffalo for the first time and asking what "insects" they were.

Throughout the article, we have formed some insights about tool perspectives, fusion, and graphical spaces that characterize Eleanor's and Dina's emerging resources and patterns of significance.

In their initial work with the motion detector, Eleanor and Dina focused on grasping the motion detector's responsiveness to their body motions, striving to identify aspects of body motion that the tool is sensitive to as well as how those aspects become graphically noticeable. We emphasize Eleanor's and Dina's treatment of the tool as a point of view from which they distinguished between significant and idiosyncratic qualities, that is, between those attributes that control the creation of graphs (e.g., getting closer to or farther away from the tower) and those that one has to comply with to achieve a good performance (e.g., keeping the button oriented toward the tower). We note that, in the context of using the tool as a point of view, Eleanor and Dina expressed the emergence of a sense of logical necessity, that enabled them to predict how the graph should look and to distinguish between possible and impossible graphs.

A conspicuous trait of Eleanor's and Dina's fusion experience was the interplay between the graph as a shape and the graph as a response to actions. We saw these shifts back and forth between these two perspectives embedded in their language when they described their plans or reviewed what motion had led to the shape of the graph. Our analysis underscores instances in which Eleanor and Dina embedded in their narratives descriptions of body actions and of the developing graphs as travels along trajectories. Fusion also involved the interplay between graphing and drawing. Treating graphs as drawings was a fruitful source of ideas and language for Eleanor and Dina. At the same time, when they attempted to use the motion detector to draw shapes, they encountered constraints that made "easy" drawings, such as a square, impossible to graph.

We characterize the graphical spaces experienced by Eleanor and Dina as lived-in spaces populated by kinesthetic and visual patterns, graphical responses, intentions, and past and present conversations. We also emphasize that Eleanor and Dina perceived graphical height not primarily as a detached variable but as indexing their experience with the tool.

Overall, our conclusions nourish views according to which learning graphing takes place in the broadening and enriching of students' lived-in spaces, in the growth of novel forms of fusion, and in the manifold development of tool perspectives. We think that these conclusions are relevant in a number of areas. In relation to the ethnographic studies on practitioners of science and technology (Goodwin, 1995; Ochs et al., 1994), they suggest that the difference between expert and lay graphers is not a difference in kind but part of a continuum. Our conclusions also invite the notions that symbolizing is a movement toward indexing human experience, that playfulness is a crucial aspect of the creation and interpretation of symbolic expressions, and that projecting anthropomorphic

qualities on tools nurtures the weaving together of logical necessity and empirical evidence.

### Classroom Implications

Some readers might argue that Eleanor's and Dina's ideas and approaches are not relevant for the teaching of graphing in schools because they are based on individual interviews that are very different from the context of classroom and group work. Our belief that Eleanor's and Dina's ideas are relevant for school teaching is based on both our experience in classrooms, where we have developed and tested a number of curricular units involving graphing (Tierney, Nemirovsky, & Noble, 1996; Tierney, Nemirovsky, & Weinberg, 1995; Tierney, Weinberg, & Nemirovsky, 1995; Wright, Nemirovsky, & Tierney, 1997), and our teaching interviews with elementary, high school, and college students. We have noticed in all settings frequent instances of students describing trips on a graphical plane as they unfold narratives that involve simultaneously events and visual attributes of the graph. We often observe students exploring tools anew when the context of use has changed or high school students wondering, for example, why the "line keeps going to the right even when [they] don't move the button."

Although a comprehensive analysis of approaches to teaching graphing is beyond the scope of this article, we illustrate how we see each theme in relation to central instructional issues. Our intent is to highlight relevant aspects of classroom activities in the light of the ideas emerging from this article. To this end, we briefly relate experiences from a fifth-grade class that piloted activities—included in the *Investigations* Patterns of Change unit (Tierney et al., 1996)—that connect the theme of tool perspectives to students playing at being the tool, the theme of fusion to students' invention of representational systems, and the theme of graphical spaces to students indexing domains of experience in the classroom.

### Tool Perspectives

Our analysis of Eleanor's and Dina's initial uses of the motion detector has led us to describe what one learns by experimenting with a tool as "the development of a tool perspective." Although one tends to ascribe this perspective to the tool itself (e.g., what the tool can see), it is a perspective that the tool user strives to enact. The tool user tries to shift aspects that she perceives as significant in the workings of the tool to the foreground and to relegate idiosyncrasies that must be complied with to the background. In so doing, a sense of logical necessity, what "should" and "should not" happen, becomes part of her emerging perspective. One way in which such a process can take place in the classroom is by designing activities in which students play at being the tool.

Our example comes from a fifth-grade class that used a computer simulation displaying a girl and a boy running along parallel tracks.<sup>14</sup> Students write LOGO commands that set the boy's and girl's step size, start position, and other parameters. After students had set the parameters of the boy's and the girl's run, we asked: "Who is going to win the race?" Some students responded to that question only after looking very attentively at the boy and the girl running on the computer screen, as if the only way to know was to carefully observe their motion. We realized that these students relied on an empirical test but did not yet envision a logical necessity for the boy and the girl to move in certain ways as determined by their commands.

Our sense was that their issue was not how to infer the result of a race out of its parameters (e.g., they knew that if runners start together, the winner is the one who runs fastest), but how to grasp the nature of the computer simulation: what the software was actually doing. In response to this realization, we designed activities using Cuisinaire rods and a meter stick for students to use in parallel with the computer simulation. Each side of the meter stick was a "track," and the Cuisinaire rods represented the step size of the boy and the girl. Students marked each step by advancing discretely each Cuisinaire rod and generated number tables by annotating the locations of the front of the Cuisinaire rods on the meter stick. Students created different motion stories (e.g., the girl started way behind the boy but she got to the end sooner) with the computer simulation and the Cuisinaire rods. Many students were able to make better sense of the simulation after acting out the motion stories with the Cuisinaire rods.

This experience helped us to become aware of the complexities inherent in supporting students' development of rich tool perspectives in which they come to foresee necessity. Playing at being the tool is a way to develop a new point of view through which one learns to enact an interpretive framework so that the tool becomes informative and enlightening.

## Fusion

We have characterized fusion as acting, talking, and gesturing without distinguishing between symbols and referents. We have exemplified how fusion involved Eleanor and Dina using their life experiences (e.g., drawing, following trajectories) to figure out what the graphs meant to them. Instead of ascribing normative and conventional meaning to a graph, our analysis suggests that, through fusion, people create meanings in ways that, at times, are consistent with cultural assumptions and, at other times, are not. From an instructional point of view, this conclusion suggests that learning norms and conventions is part of the broader process of bringing

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<sup>14</sup>The software is called Trips. It has been jointly developed with Douglas Clemets and Julie Sarama. It is published together with the Patterns of Change unit (Tierney et al., 1996).

students' own experience to bear. An approach that acknowledges this embedded role of norms and conventions is the introduction of graphical conventions in the context of students inventing their own representational systems. Other researchers have explored similar ideas (Bednarz, Dufour-Janvier, Poirier, & Bacon, 1993; diSessa et al., 1991; Godfrey & O'Connor, 1995; Krabbendam, 1982). Students' invention of representational systems are important, not because we would expect that they will necessarily develop the mathematical ideas in question spontaneously and autonomously but because these activities can generate a context for a meaningful encounter with them.

For example, the fifth-grade students represented ways of walking along a straight line and gave these representations to other students for them to interpret and walk. A trip that Lin and her partner, Tyra, intended to communicate was broken down into five segments: 3 baby steps, stop for 5 sec, 4 big steps, stop for 3 sec, 15 baby steps. They showed step size by pictures of large and small shoes, stopping with a full hand, and time with a certain number of fingers up and numbers written near them.

Their representation is not immediately related to distance or velocity graphs. However, there are important elements in it that are central to such graphs: the size and number of steps, how the step size changes over time and space, how much time to wait during stops, and so forth. By creating graphical approaches to solving problems before conventions are introduced, students become aware of the elements that they need to communicate in their representations. Students' encounters with ways that conventional ideas differ from systems they have invented help them become aware of graphical conventions in the context of their own need to communicate. Conventions, then, are techniques for communication and analysis within a broad domain of graphical ideas and activities.

## Graphical Space

We have characterized graphical spaces as lived-in spaces in which the graph indexes a particular domain of experience (e.g., moving along a straight line at varying speeds). One major implication for the design of classroom activities is that one needs to pay close attention to how teachers and students conceive of both the domain of experience and the domain of indexing approaches that are being discussed. The following examples illustrate ways in which domains of experience and indexing approaches were redefined.

In the Patterns of Change Unit, students create number tables describing successive positions of a student walking along a 10-m tape measure, marked by dropping small bean bags at regular intervals of time. Students discuss different patterns of bean bags on the tape measure, how they correspond to different speeds of walking, and how number tables can indicate those patterns.

On negotiating a domain of experience, we struggled with how to focus students on changes of speed in particular. For example, in walking along a straight line there are many aspects one could pay attention to (walking forward and backward, on tiptoes, jumping, with arms up or down, etc.). However, jumping could actually be a way of going fast; walking on tiptoe might be a meaningful way to accent slowness. Telling students to think only of changes in speed does not establish an agreement as to what is pertinent and what is not. “Fuzzy” boundaries can be useful in engaging with students in a gradual process of creating a mutually agreed on domain of experience that preserves the focus of curricular goals and is enriched by students’ contributions.

On the domain of indexing approaches, after a few classroom sessions, we came to realize that the notion of marking positions by dropping bean bags at regular intervals of time, no matter how natural it seemed to us, is a very particular way to proceed. Some students tended to mark where the speed changes (e.g., “from this bean bag to that one, go at two steps a second”); others used the bean bags to indicate step size, so that each step is indicated by the distance between two successive bean bags. These approaches led us to make crucial changes in the curricular design. We realized that step size is an extremely useful variable because it unifies time and space; that is, the bean bags marking steps given at a regular interval of time (e.g., every 2 sec) indicate simultaneously the “when” and the “where” of the walk.

In sum, we think that the ideas of this article have a number of classroom implications. Among them, we point out the following:

1. Playing at being a tool is a rich resource for using technology in learning graphing.
2. Children’s inventions of representational systems to communicate stories and ideas are fundamental for the meaningful introduction of conventions and socially established approaches (e.g., regular scale, a time axis).
3. The interplay between discrete and continuous representations (e.g., patterns of dropped bean bags and continuous graphs) is a suitable theme for learning about graphing at all school levels.
4. Representing situations of motion (e.g., walking along a path, driving a handheld car or electric train) in a variety of ways allows students to encounter ideas such as distance, speed, time, and acceleration.
5. Fusion of physical action and graphical shapes is a major resource to engage students in conversations around the production and the interpretation of graphs.
6. The definition of the domains of experience and of indexing approaches that are to become part of classroom activities should emerge from the ongoing negotiation between teachers and students.

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